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Lubrication of Nonconformal Contacts

Yeau-Ren Jeng
Lewis Research Center
Cleveland, Ohio



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NOMENCLATURE

a	semimajor axis of contact ellipse, m
B	basic speed ratio, δ/S
b	semiminor axis of contact ellipse, m
D	influence coefficient matrix
D	percentage difference, $[(H_0 - H_0)/H_0]*100$
d	elastic deformation, m
d_b	ball diameter, m
d_m	pitch diameter, m
E	modulus of elasticity, N/m
E'	effective elastic modulus, $2/[1 - \nu_a^2)/E_a + (1 - \nu_b^2)/E_b]$, N/m
τ	shear force per unit length, N/m
G	dimensionless materials parameter, $E'/p_1\nu$, as
H	dimensionless film thickness, h/R_x
H_0	dimensionless central film thickness, h_0/R_x
H_0	calculated dimensionless central film thickness from least-square analysis
h	film thickness, m
h_0	central film thickness (minimum film thickness as well in piezoviscous-rigid regime of lubrication), m
h_{min}	minimum film thickness, m
H_{min}	dimensionless minimum film thickness, h_0/R_x
k	ellipticity parameter, b/a
P	dimensionless pressure, p/E'
p	pressure, N/m ²

$p_{iv,as}$	asymptotic isoviscous pressure, N/m^2
p_s	pressure spike, N/m^2
R	effective radius, m
r	radius of curvature, m
S	geometrical distance, m
s, t	constants defining fluid used in equation (2.7)
U	dimensionless entrainment velocity, $n_0 u / E' R_X$
u_x	mean entrainment velocity in x direction, m/s
u_{ax}	surface velocity of solid a in x direction, m/s
v	slip velocity, m/s
v_x	slip velocity in x direction, m/s
W	dimensionless load parameter, $w / E' R^2$
w	load, N
X, Y	dimensionless coordinate, $x / R_X, y / R_X$
x, y	dimensionless coordinate, $x / b, y / a$
Z	viscosity pressure index, a dimensionless constant
z	height of surface a, m
z_a	mean height, $(z_a + z_b) / 2$
α	radius ratio, R_y / R_X
γ	roughness anisotropy index
δ_a	random roughness height of surface a, $z_a - z_a^*$, m
η	lubricant viscosity, $(N \cdot s) / m^2$
η	dimensionless viscosity, η / η^0
η_0	lubricant viscosity at atmospheric pressure, $(N \cdot s) / m^2$
θ	angle between lubricant entrainment vector and the x direction

μ	coefficient of rolling friction
ν	Poisson's ratio
ρ	lubricant density, kg/m ³
ρ	dimensionless density, ρ/ρ_0
ρ_0	lubricant density at atmospheric pressure, kg/m ³
σ	standard deviation of $(\delta_b - \delta_a)$, m
ϕ	$\rho h^3/2$
ϕ^D	pressure flow factors
ϕ^S	shear flow factors
Λ	dimensionless film parameter, h^*/a

Subscripts

a	solid a
b	solid b
x,y	coordinates in plane of lubricating film

Superscripts

*	ensemble average (expectation) operator for stochastic quantity
	vector quantity

Chapter 1

INTRODUCTION

1.1 Conformal and Nonconformal Contacts

In many contacts between machine elements, forces are transmitted through thin, but continuous, fluid film. The fluid film lubrication is generally considered the ideal form of lubrication since the absence of asperity interaction of the surfaces provides low friction and high resistance to wear. The fluid film lubrication as related to hydrodynamic lubrication in journal and thrust bearings exhibits conformal surfaces, i.e., the surfaces snugly fit into each other with a high degree of geometrical conformity, as shown in figure 1.1. and the load is carried over a relatively large area. The load-carrying surface area remains essentially constant while the load is increased. The minimum film thickness in a hydrodynamically lubricated bearing is a function of applied load, speed, lubricant viscosity, and geometry.

Many machine elements have contacting surfaces that do not conform to each other, as shown in figure 1.2 for a rolling-element bearing. The full burden of the load is carried by a small contact area, the contact area between nonconformal surfaces enlarges with increasing load, but it remains small compared with the contact area between conformal surfaces. The form of lubrication normally found in nonconformal contacts is elastohydrodynamic lubrication.

The load per unit area in conformal bearings is relatively low, typically only 1 MN/m^2 and seldom over 7 MN/m^2 . The load per unit area in nonconformal contacts generally exceeds 700 MN/m^2 even at modest applied load. These high pressures result in elastic deformation of the bearing materials such that elliptical contact areas are formed for oil film generation and load support. High contact pressures can produce considerable increases in fluid viscosity. Inasmuch as viscosity is a measure of a fluid's resistance to flow, this increase greatly enhances the lubricant's ability to support load without being squeezed out of the contact zone. The high contact pressures between nonconformal surfaces result in an elastic deformation of the surfaces and large increases in the fluid's viscosity. The resulting minimum film thickness is a function of the parameters found for hydrodynamic lubrication with the addition of an effective modulus-of-elasticity parameter and a pressure-viscosity coefficient.

1.2 Lubrication Regimes

Since the lubrication of nonconformal contacts is influenced by two major physical effects, the elastic deformation of the solids under an applied load and the increase in fluid viscosity with pressure, four main regimes of fluid film lubrication have been delineated depending on the relative magnitude of these effects, (Hamrock and Dowson, 1981):

Isoviscous-rigid. - In this regime, the magnitude of the elastic deformation of the surface is an insignificant part of the fluid film thickness and the maximum pressure in the contact is too low to increase fluid viscosity significantly. This form of lubrication is typically encountered in circular-arc thrust bearing pads; and in industrial coating processes.

Piezoviscous-rigid. - In this regime, the pressure within the contact is sufficiently high to increase the fluid viscosity within the conjunction significantly, while the deformation of the surfaces remains an insignificant part of the fluid film thickness. This form of lubrication encountered on roller flanges, in contacts in moderately loaded cylindrical tapered rollers, and between some piston rings and cylinder liners.

Isoviscous-elastic. - In this regime, the elastic deformation of the solid is a significant part of the fluid film thickness, but the pressure within the contact is quite low and insufficient to cause any substantial increase in viscosity. This situation arises with materials of low elastic modulus, and it is a form of lubrication that may be encountered in seals, human joints, tires, and elastomeric-material machine elements. This lubrication regime is sometimes referred to as soft EHL.

Piezoviscous-elastic. - In fully developed elastohydrodynamic lubrication, the elastic deformation of the solids is often a significant parts of the fluid film thickness, and the pressure within the contact is high enough to cause a significant increase

in the viscosity of the lubricant. This form of lubrication is typically encountered in ball and roller bearings, gears and cams. This lubrication regimes is also referred to as hard EHL.

1.3 Historical Developments

The theory of fluid film lubrication developed by Osborne Reynolds (1886) was first applied to nonconformal contacts by Martin (1916). This study represented an early attempt to explain the mechanism of spur gear lubrication and Martin's celebrated solution considered the lubrication of rigid cylindrical solids by means of an isoviscous, incompressible fluid. Martin's solution greatly underestimates the film thickness; however, it was a useful beginning to the theoretical study of elastohydrodynamic lubrication of line contacts. Some 30 years transpired before any significant accomplishments were made in solving the EHL line-contact problem. Ertel (1984) (who used to be referred to as Grubin (1949), now the record is straight after a 40-year-old wrong (Cameron, 1985)) obtained the first satisfactory solution to this problem by taking account of elastic distortion and viscosity-pressure effects. In Ertel's analysis it was assumed that the shape of the elastically deformed solids in a highly loaded lubricated contact is the same as the shape produced in a dry Hertzian contact. Ertel's approach produced an excellent account of the physical mechanism of the lubrication process in highly loaded EHL line contacts.

Not until the middle years of the present century has most of the work on elastohydrodynamic lubrication dealt with line contacts. A remarkable extension of the classical hydrodynamic lubrication theory for nonconformal contacts was recorded by Kapitza (1955), who presented an elegant analysis of both line and point contacts lubricated by either isoviscous or piezoviscous fluids. However, applying the half-Sommerfeld boundary condition used in Kapitza's analysis violates flow continuity at the cavitation boundary. Archard and Cowking (1965-1966) developed an elastohydrodynamic theory for point contacts of the form encountered between two spheres and thus extended the classical study of Kapitza as Ertel had extended that of Martin. The Hertzian contact zone was assumed to form a parallel film region, and the generation of high pressure in the approach to the Hertzian zone was considered. The results of the Archard and Cowking analysis led to the concept of a side-leakage factor, which represents the proportional reduction in pressure attributable to side leakage. Only in the 1970's did the complete numerical solution of the isothermal elastohydrodynamical lubrication of point contacts successfully emerge. In the years 1974 to 1978, Hamrock and Dowson (1976a, 1976b, 1977a, 1977b, 1978, 1979a, 1979b) published eight papers on EHL lubrication. These papers gave a complete approach to the solution of the elastohydrodynamic lubrication problem for point contacts. Their numerical solutions representative of materials of both high and

low elastic modulus and this enabled them to write expressions for minimum film thickness in both the isoviscous-elastic and piezoviscous-elastic regimes of lubrication as follows:

Isoviscous-elastic

$$H_{min} = 7.43 U^{0.65} W^{-0.21} (1 - 0.85e^{-0.31k}) \quad (1.1)$$

Piezoviscous-elastic

$$H_{min} = 3.63 U^{0.68} G^{0.49} W^{-0.073} (1 - e^{-0.68k}) \quad (1.2)$$

Brewe et.al. (1979) have obtained a film thickness equation for the lubrication of fully flooded, rigid, isoviscous point contacts through a numerical analysis that used a more realistic Reynolds condition for the film rupture in the exit region. It was found that the minimum film thickness had the same speed, viscosity, and load dependence as Kapitza's classical solution. However, the incorporation of the Reynolds boundary condition resulted in an additional geometry effects. The results can be written as:

Isoviscous-rigid

$$H_{min} = 128\alpha \left\{ \frac{1}{1 + \frac{(2/3)\alpha}{W}} U [0.131 \tan^{-1} \left(\frac{\alpha}{2} \right) + 1.683] \right\}^2 \quad (1.3)$$

1.4 The Objective of Present Work

Equations (1.1), (1.2), and (1.3) thus represent the minimum film thickness formula for three of the four regimes of lubrication of nonconformal contacts. However, full solutions

have not been available for the regime of piezoviscous-rigid lubrication with the same measure of confidence.

It is the first purpose of this work to study the piezoviscous-rigid regime of lubrication and to present results which will carry the same measure of confidence as those already available for the other three regimes of lubrication. This will not only enable the minimum film thickness to be calculated with greater accuracy in the piezoviscous-rigid regime, but will also complete the map of lubrication regimes of nonconformal contacts. The extended set of solutions includes the geometric effect where radius ratio less than one and the influence of lubricant entrainment direction which are not presented in equations (1.1), (1.2), and (1.3) (Jeng, et al., 1985). The results of the present work enable the lubricant film thickness to be predicted with increased confidence for a wide range of machine elements.

Over the last 30 or so years, major strides have been made in the understanding of the mechanism of fluid lubrication in nonconformal machine elements as reviewed above. To obtain a better understanding of the failure mechanism in machine elements, the next generation of elastohydrodynamic lubrication analyses should incorporate such effects as non-Newtonian fluids, surface roughness, and temperature. The second contribution of this work is the development of an improved numerical method of calculating elastic deformation in contact stresses. Incorporating an accurate elastic deformation calculation in the EHL numerical

scheme is necessary to reach the higher upper range of EHL parameters as indicated in Houpert and Hamrock (1985) and provides a means to study surface roughness effects on elastohydrodynamic lubrication and micro-elastohydrodynamic lubrication. A piecewise biquadratic polynomial is used to approximate the pressure distribution on the whole domain analyzed. The deformation of every node is expressed as a linear combination of the nodal pressures whose coefficients can be combined into an influence coefficient matrix. It has been proved that this approach has higher numerical accuracy and smaller computer storage size for the influence coefficient matrix.

This improved elasticity calculation is successfully incorporated into the EHL numerical scheme. Using this revised numerical technique, the surface roughness effects on the elastohydrodynamic lubrication of point contact is studied as an attempt to extend the ideal elastohydrodynamic lubrication model to the real bearing systems. This is the third major feature of this study. Surface roughness effects play a significant role in determining the fatigue life of nonconformal contact lubrication. The introduction of surface roughness effects by a study of Reynolds equation with random variables constitutes a fundamental contribution to stochastic continuum mechanics.

The analysis is based on a Reynolds equation modified through surface topography. Within the approximation of the second order perturbation approach to stochastic flow, only two parameters

govern the pressure developed by the film. In dimensionless form these are Λ , the ratio of nominal film thickness to rms roughness amplitude and γ , the ratio of the correlation lengths parallel and perpendicular to the lay direction of the surface texture. Conditions typical of an EHL contact in the piezoviscous-elastic regime entrained in pure rolling on rough bearing are demonstrated. The results are compared with smooth surface solutions.

Furthermore, the fourth phase of this work consists of experiments designed and conducted to study the transient EHL effects in instrument ball bearings. A parched subregime of elastohydrodynamic lubrication, lying between starved and mixed, is proposed to describe results of these experiments. A system is parched if oil films outside the Hertzian contacts are so thin they do not flow under service accelerations. Parched operation requires the least driving torque and provides the best spin definition possible of any lubrication regime. The results are directly applicable to other nonconformal contacts such as gears, roller bearings, etc.

Chapter 2

RELEVANT EQUATIONS

The study of fluid film lubrication is, from a mathematical standpoint, the application of a reduced form of the Navier-Stokes equation in association with the continuity equation. The resulting differential equation was formulated by Reynolds (1886) in the wake of a classical experiment by Tower (1883) in which the existence of a thin fluid film was detected from measurement of pressures within the lubricant.

The Reynolds equation contains viscosity and density terms. These properties of the lubricants depend on pressure when isothermal conditions are considered; hence it is necessary to deal with the pressure-viscosity and pressure-density formula. The Reynolds equation also contains the film thickness as a parameter. The film thickness is a function of the geometry and the elastic behavior of the contacting solids. The governing equations that describe fluid film lubrication will be developed in this chapter.

2.1 Reynolds equation

The equations of motion for a Newtonian fluid are known as the Navier-Stokes equations. They are derived by applying to a fluid the principles of conservation of momentum. The basic equation of fluid-film lubrication, the Reynolds equation, can be derived from the reduced form of the Navier-Stokes equation (representing the conservation of momentum) and continuity

equations (representing the conservation of mass). A general form of the Reynolds equation can be established in the restriction of the following assumptions:

(1) Inertia and body force terms are negligible compared with the pressure and viscous terms, i.e. $(\rho gh^2/\eta u) \ll 1$ and $((\rho u h/\eta)(h/l)) \ll 1$ where u is a characteristic fluid velocity, h is characteristic of the film thickness, η is the characteristic viscosity, ρ is the characteristic density, and l is a characteristic length in the flow direction and g is gravity.

(2) The radius of curvature of the solids bounding the oil film is large compared with the thickness of the lubricant film, $h/R \ll 1$.

The detail derivations can be found in Dowson (1962) or Hamrock and Dowson (1982). The general form of the Reynolds equation can be referred to cartesian coordinates as follows:

$$\frac{\partial}{\partial x} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial y} \right) = \frac{\partial}{\partial x} \left[\frac{\rho(u_{ax} + u_{bx})h}{2} \right] + \frac{\partial}{\partial y} \left[\frac{\rho(u_{ay} + u_{by})h}{2} \right] + \frac{\partial}{\partial t} (\rho h) \quad (2.1)$$

where

u_{ax} surface velocity of solid a in x direction

u_{bx} surface velocity of solid b in x direction

u_{ay} surface velocity of solid a in y direction

u_{by} surface velocity of solid b in y direction

If the surface velocities are assumed to be a constant, thus the Reynolds equation for steady state conditions can be expressed as:

$$\frac{\partial}{\partial x} \left(\frac{\rho h^3}{\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\rho h^3}{\eta} \frac{\partial p}{\partial y} \right) = 12 \left[u_x \frac{\partial(\rho h)}{\partial x} + u_y \frac{\partial(\rho h)}{\partial y} \right] \quad (2.2)$$

where

$$u_x = (u_{ax} + u_{bx})/2$$

$$u_y = (u_{ay} + u_{by})/2$$

By introducing u and θ , where $u = \sqrt{u_x^2 + u_y^2}$,

$\theta = \text{TAN}^{-1} (u_x/u_y)$ equation (2.2) becomes

$$\frac{\partial}{\partial x} \left(\frac{\rho h^3}{\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\rho h^3}{\eta} \frac{\partial p}{\partial y} \right) = 12u \left[\cos \theta \frac{\partial(\rho h)}{\partial x} + \sin \theta \frac{\partial(\rho h)}{\partial y} \right] \quad (2.3)$$

2.2 Pressure-Viscosity Formula

It is generally known that the viscosity of a lubricant is a function of temperature and pressure and a generally accepted relation which describes the viscosity-pressure dependency is the Barus (1893) equation:

$$\eta = \eta_0 e^{\alpha p} \quad (2.4)$$

where

η viscosity at gage pressure

η_0 viscosity at atmosphere pressure

α pressure-viscosity coefficient of lubricant

Unfortunately, pressure-viscosity data seldom follow this simple relation, and it is valid as a reasonable approximation only in a moderate-pressure range.

Roelands (1966) in a more extensive experimental study of the effect of pressure on the viscosity of lubricants has developed an empirical formula written as:

$$\log \eta + 1.200 = (\log \eta_0 + 1.200) \left(1 + \frac{p}{2000}\right)^Z \quad (2.5)$$

where

p gauge pressure kgf/cm²

Z viscosity index, a dimensionless constant

Rearranging terms gives:

$$\eta = \eta_0 (1 + p/2000)^Z \times 10^{1.2[1 + (p/2000)^Z - 1]} \quad (2.6)$$

The temperature effect is normally accounted for in η_0 .

2.3 Pressure-Density Formula

The variation of density with pressure is roughly linear at low pressures, but the rate of increase falls away at high pressures. From Dowson and Higginson (1966), the dimensionless density for mineral oil can be written as

$$\frac{\rho}{\rho_0} = 1 + \frac{sp}{1 + tp} \quad (2.7)$$

where s and t are constants that depend on the fluid

2.4 Elasticity Equation

From Timoshenko and Goodier (1951), the elastic deformation at a point (x, y) of a semi-infinite solid subjected to pressure p at the point (x', y') can be written as

$$d(d) = \frac{2p \, dx' \, dy'}{\pi E' r} \quad (2.8)$$

The elastic deformation at a point (x', y') due to the pressure over the domain is thus

$$d = \frac{2}{\pi} \iint_A \frac{dx' dy'}{\left[(y - y')^2 + (x - x')^2 \right]^{1/2}} \quad (2.9)$$

The evaluation of this equation will be discussed more fully in Chapter 4.

2.5 Film Shape

The separation of two rigid solids a and b having radii of curvature (r_{ax}, r_{ay}) and (r_{bx}, r_{by}) in the vicinity of the point of closest approach can be considered as geometrically equivalent to a solid of principal radii (R_x, R_y) adjacent to a plane as shown in figure 2.1. The geometrical requirement is that the separation of the ellipsoidal solids in the initial and equivalent situation should be the same at equal values of x . The effective radii of curvature can be expressed as

$$\begin{aligned} \frac{1}{R_x} &= \frac{1}{r_{ax}} + \frac{1}{r_{bx}} \\ \frac{1}{R_y} &= \frac{1}{r_{ay}} + \frac{1}{r_{by}} \end{aligned} \quad (2.10)$$

It is assumed that convex surfaces exhibit positive curvature and concave surfaces negative curvature. If the center of curvature lies within the solid, the radius of curvature is positive; if the center of curvature lies outside the solid, the radius is negative.

The separation of two rigid solids (fig. 2.1(a)) in which the principal axes of inertia of the two bodies are parallel can be written as

$$S = S_{ax} + S_{bx} + S_{ay} + S_{by} \quad (2.11)$$

where

$$S_{ax} = r_{ax} - \sqrt{r_{ax}^2 - x^2}$$

$$S_{bx} = r_{bx} - \sqrt{r_{bx}^2 - x^2}$$

$$S_{ay} = r_{ay} - \sqrt{r_{ay}^2 - y^2}$$

$$S_{by} = r_{by} - \sqrt{r_{by}^2 - y^2}$$

The separation in terms of the coordinate and the effective radius of curvature (fig. 2.1(b)) is

$$S(x,y) = R_x - \sqrt{R_x^2 - x^2} + R_y - \sqrt{R_y^2 - y^2} \quad (2.12)$$

Thus, the film thickness between two rigid bodies in point contact can be written as

$$h = h_0 + S(x,y) \quad (2.13)$$

If elastic deformation is taken into account, the film thickness can be expressed as

$$h = h_0 + S(x,y) + d(x,y) \quad (2.14)$$

Chapter 3

PIEZOVISCOUS-RIGID REGIME

In the piezoviscous-rigid regime of lubrication, Marko et al. (1979) proposed a formulae obtained by means of curve fitting of Dowson and Whitaker's (1965) results for line contacts. In the case of point contact, Hamrock and Dowson (1978) proposed an interim measure that Blok's (1952) solution for line contacts might be adjusted by the application of the same "side-leakage factor" as that derived for piezoviscous-elastic condition. Recently, the numerical solution for a compressible Newtonian lubricant exhibiting pressure-viscosity characteristic and subject to Reynolds cavitation boundary condition was obtained by Dowson et al. (1983). The formulae from the last two papers describe the "limiting" film thickness generates infinite pressures, as discussed by Blok (1952). However, they do not include the load parameter W , which has a strong effect on film thickness when piezoviscous effects are considered. Houpert (1984) developed a sophisticated general formula by means of curve fitting of Dalmaz's (1979) results. A shortcoming of Dalmaz (1979) work is that he used the Barus exponential formula for pressure-viscosity characteristics which tends to give high values of viscosity compared to results obtained from Roelands formula (Jones, et al. (1975)). Furthermore, Dalmaz's (1979) results for the isoviscous case produced a lower exponent on W/U than Brewe et al. (1979). This appears to be due to starvation effects entering the fully

flooded results from the designation of the inlet boundary condition.

Figure 3.1 obtained from Meuleman et al. (1985) shows the differences between the Barus formula, Rolands formula and the experimental data of Hirst and Moose (1978). It is apparent that the Rolands formula represents the experimental result for the pressure-viscosity relationship more accurately than the Barus formula at high pressures, and the Rolands formula is used in this study. The researchers in the past used the Barus formula and neglected the lubricant compressibility to obtain a straight forward numerical analysis to the resulting linear partial differential equation. The numerical analysis employed in this study is more complicated because the resulting partial differential equation is now nonlinear. But this nonlinear equation is more realistic.

In the current study, the numerical solution for the piezoviscous-rigid regime of lubrication is presented for the full spectrum of conditions. The influence of lubricant entrainment direction has also been studied. The condition that the lubricant entraining vector is not parallel to the minor axis of the contact might arise, for example, in helical, spiral bevel, and hypoid gears. The radius ratio is varied from 0.2 to 64, to cover any contact ranging from something similar to a disk rolling on a plate ($\alpha < 1$) to a contact approaching a nominal line contact ($\alpha \rightarrow 70$) such as a barrel-shaped roller against a plate. The

influence of the dimensionless speed, load, and materials parameter on minimum film thickness are also investigated. The dimensionless load parameter is varied over a range of an order of magnitude. The dimensionless speed parameter is varied over a range 5.6 times the lowest speed value. Conditions corresponding to the use of solid materials of steel, bronze, and silicon nitride and lubricants of paraffinic and naphthenic mineral oils are considered in obtaining the exponent in the dimensionless materials parameter. Forty-one cases are used to obtain a simple empirical minimum film thickness formula. Contour plots are shown that indicate in detail the pressure developed between the solid.

3.1 Method of Calculation

By introducing the following dimensionless groups

$$\bar{X} = \frac{X}{R_x}, \quad \bar{Y} = \frac{Y}{R_x}, \quad \bar{\rho} = \frac{\rho}{\rho_0}, \quad \bar{\eta} = \frac{\eta}{\eta_0}, \quad H = \frac{h}{R_x}, \quad P = \frac{p}{E'}$$

to the equations described in Chapter 2 the set of dimensionless equations that describe the piezoviscous-rigid regime of lubrication can be expressed as follows.

The Reynolds equation is

$$\frac{\partial}{\partial \bar{X}} \left(\frac{\bar{\rho} \bar{H}^3}{\bar{\eta}} \frac{\partial P}{\partial \bar{X}} \right) + \frac{\partial}{\partial \bar{Y}} \left(\frac{\bar{\rho} \bar{H}^3}{\bar{\eta}} \frac{\partial P}{\partial \bar{Y}} \right) = 12U \left[\cos \theta \frac{\partial(\bar{\rho} \bar{H})}{\partial \bar{X}} + \sin \theta \frac{\partial(\bar{\rho} \bar{H})}{\partial \bar{Y}} \right] \quad (3.1)$$

where

$$U = \eta_0 u / E' R_x$$

The pressure-viscosity formula is from Roelands (1966)

$$\bar{\eta} = \left(\frac{\eta_{\infty}}{\eta_0} \right)^{[1 - (1 + PE^1/c)^2]} \quad (3.2)$$

where

$$\eta_{\infty} = 6.31 \times 10^{-5} \text{ Ns/m}^2$$

$$c = 1.96 \times 10^8 \text{ Ns/m}^2$$

The pressure-density formula is from Dowson and Higginson (1966)

$$\bar{\rho} = 1 + \frac{sPE^1}{1 + tPE^1} \quad (3.3)$$

where s and t are constants that depend on the fluid.

The film thickness formula is

$$H = H_0 + \frac{S(X,Y)}{R_x} \quad (3.4)$$

where H_0 is central film thickness also minimum film thickness for rigid contacts.

Note the surfaces are considered rigid. Elastic effects will be considered in Chapter 4.

Boundary Condition

The boundary conditions are:

(a) The Reynolds boundary condition is applied in the divergent film. Namely, $P = dP/dX = dP/dY = 0$ at the cavitation boundary.

(b) The pressure on the boundaries of the computation region are assumed to be zero. The fully flooded conjunction is

considered in the present study, i.e., increasing the computation region of the conjunction does not effect the results.

Numerical Analysis

Having defined the density, viscosity and film thickness we are in a position to solve the Reynolds equation subjected to the Reynolds cavitation boundary condition. The dimensionless pressure of P is notorious for its steep gradient, which is not welcomed when performing numerical analysis via relaxation methods. In order to produce a more gentle curve, a parameter is introduced where

$$\phi = PH^{3/2}$$

This substitution also has the advantage of eliminating all terms containing derivatives of products of H and P or H and ϕ . Substituting equation (3.5) into equation (3.1), one obtains

$$\begin{aligned} H^{3/2} \left[\frac{\partial}{\partial X} \left(\frac{\bar{\rho}}{\bar{\eta}} \frac{\partial \phi}{\partial X} \right) \right] - \frac{3}{2} \phi \frac{\partial}{\partial X} \left(\frac{\bar{\rho}}{\bar{\eta}} H^{1/2} \frac{\partial H}{\partial X} \right) + H^{3/2} \left[\frac{\partial}{\partial Y} \left(\frac{\bar{\rho}}{\bar{\eta}} \frac{\partial \phi}{\partial Y} \right) \right] \\ - \frac{3}{2} \phi \frac{\partial}{\partial Y} \left(\frac{\bar{\rho}}{\bar{\eta}} H^{1/2} \frac{\partial H}{\partial Y} \right) = 12 \left[\cos \theta \frac{\partial(\bar{\rho}H)}{\partial X} + \sin \theta \frac{\partial(\bar{\rho}H)}{\partial Y} \right] \quad (3.6) \end{aligned}$$

The method of frozen coefficients (sometimes also referred to as Kacanov's method) (1975) is applied to solve this nonlinear partial differential equation. It means that the solution for the set of equations is obtained by updating the nonlinear terms using an iterative procedure until the differences between the pressure variables for successive iterations are as small as desired.

Second order central finite difference approximations are used to

the modified form of the Reynolds equation, thus forming a set of algebraic equations that are solved by the Gauss-Seidel iterative method with overrelaxation. For optimal efficiency, a variable-mesh structure (shown in fig. 3.2) is used to enhance the accuracy in the region of high pressure and large pressure gradient.

Normal Applied Load

The normal applied load can be evaluated by

$$W = \iint p(x,y) \, dx \, dy \quad (3.7)$$

The double Simpson's integration technique are applied to the integration.

A typical example of the pressure distribution in the regime of piezoviscous-rigid lubrication is shown in figure 3.3. Figure 3.3(a) is a 3-dimensional pressure distribution. Figure 3.3(b) shows the pressure profile in the center of the conjunction along the rolling direction. It is observed that the pressure profile in the regime of piezoviscous-rigid lubrication is very picky.

3.2 Results

Dimensionless Grouping

From the variables of the numerical analysis the following dimensionless groupings are written:

(1) Dimensionless Film Thickness

$$H = h/R_x \quad (3.8)$$

(2) Dimensionless Load Parameter

$$W = w/E'R_x^2 \quad (3.9)$$

(3) Dimensionless Speed Parameter

$$U = \eta_0 u / E' R_x$$

Where

$$u = \sqrt{u_x^2 + u_y^2} \quad (3.10)$$

(4) Dimensionless Materials Parameter

$$G = E' / p_{1V,as} \quad (3.11)$$

Where $p_{1V,as}$ = asymptotic isoviscous pressure gradient obtained from Roelands (1966)

(5) Radius Ratio

$$\alpha = R_y / R_x \quad (3.12)$$

(6) Lubricant Entraining Angle

$$\theta = \tan^{-1} (u_y / u_x) \quad \theta \text{ in degree} \quad (3.13)$$

The dimensionless film thickness can thus be written as a function of the other five parameters

$$H_0 = f(W, U, \alpha, G, \theta) \quad (3.14)$$

The most important practical aspects of hydrodynamic lubrication of nonconformal contacts is the determination of the minimum film thickness within the contact. Therefore, in the fully flooded results to be presented, the dimensionless parameters (W , U , α , G and θ) will be varied and the effect on the minimum film thickness will be studied. Note that in equation (3.8) through (3.12) by changing the normal applied load w the dimensionless load W is changed but the other dimensionless parameters remain unchanged. Similar statements can be made about changing u in

the dimensionless speed U , and R in the radius ratio.

Influence of Load

The dimensionless parameters U , G , α and θ are kept constant while varying the dimensionless minimum film thickness H_0 to get the dimensionless load capacity W at each different H_0 . The values at which the remaining parameters U , G , α and θ are held constant during the calculations are $U = 0.16833 \times 10^{-11}$, $G = 4522$, $\alpha = 16$, and $\theta = 0$.

Table 3.1 shows the computed values of load capacities for 10 values of minimum film thickness. From these 10 pairs of data, it is possible to determine a good empirical relationship between the minimum film thickness H_0 and the load capacity W :

$$H_0 = C_1 W^{C_2} \quad (3.15)$$

By applying a least-square power fit to the 10 pairs of data $[(W_i, H_{0i}), i = 1, 2, \dots, 10]$, the values of C_1 , C_2 are found to be $C_1 = 2.60615 \times 10^{-12} \approx 2.606 \times 10^{-12}$ $C_2 = -0.88019 \approx -0.880$. From the value of C_2 and equation (3.15), the effect of load on minimum film thickness is written as

$$H_0 \propto W^{-0.880} \quad (3.16)$$

In figure 3.4 the variation of dimensionless film thickness with dimensionless load is plotted for 10 data points. A linear plot is observed when plotted on log-log scale. The percentage difference between the minimum film thickness obtained from

computational results (H_0) and the minimum film thickness from the least square fit equation (\tilde{H}_0) is expressed as

$$\tilde{D} = [(\tilde{H}_0 - H_0)/H_0] * 100 \quad (3.17)$$

Influence of Speed

If the surface velocity u is changed the dimensionless speed parameter U is modified as shown in equation (3.10), but the other dimensionless parameters (H , G , α and θ) remain constant. The values at which these dimensionless parameters were held constant in the calculations performed to determine the influence of speed on film thickness are

$$\alpha = 16, \quad H_0 = 4.8 * 10^{-6}, \quad G = 4522, \quad \theta = 0$$

Values of the dimensionless speed parameter U and the corresponding dimensionless load capacities as obtained from the numerical computations are presented in table 4.2. Since the relationship between the minimum film thickness H_0 and the load capacity W has been obtained. The relationship between minimum film thickness and speed parameter can be written in the form as

$$H_0/W^{-0.88} = C_3 U^{C_4} \quad (3.18)$$

By applying a least-square fit to the 6 pairs of data $[(U_i, H_{0i})]$, $i = 1, \dots, 6$, the values of C_3 and C_4 are found to be $C_3 = 1.4899 * 10^{-13} \approx 1.49 * 10^{-13}$ and $C_4 = 1.2655 \approx 1.266$. From the value obtained for C_4 and equation (3.18), the effect of dimensionless speed on dimensionless film thickness can be written as

$$H_0 \propto U^{1.266} \quad (3.19)$$

The data presented in table 3.2 is presented in figure 3.5. As was true for the load versus film thickness results the speed versus film thickness are linear when plotted on a log-log scale.

Influence of Radius Ratio

In order to study the effect of geometry on minimum film thickness, the dimensionless parameters for minimum film thickness H_0 , speed U , materials parameter G , and lubricant entrainment direction are held constant at the following values:

$$H_0 = 4.8 \times 10^{-6}, \quad U = 0.1683 \times 10^{-11}, \quad G = 4522, \quad \theta = 0$$

Table 3.3 shows the computed values of dimensionless load capacity W for 22 values of radius ratio. It is possible to determine a good empirical relationship between the H and for the condition considered in the computation. The form of relationship chosen after investigating a number of different expressions can be written as

$$1 - (H_0/H_{0,r}) = \tilde{A}e^{\tilde{B}} \quad (3.20)$$

where $H_{0,r}$ is chosen to be the film thickness at rectangular contact.

A least-square exponential curve was fitted to the 22 pairs of data points to obtain values for \tilde{A} and \tilde{B} in equation (3.20). The values of \tilde{A} and \tilde{B} in equation (3.20) obtained from the least-square fit are $\tilde{A} = 0.989 \approx 1.00$ and $\tilde{B} = 0.03866 \approx$

0.0387. Substituting these values of \tilde{A} and \tilde{B} into equation (3.20) the following relationship between the radius ratio and minimum film thickness results,

$$H_0 \alpha (1 - e^{-0.0387 \alpha}) \quad (3.21)$$

The effect of radius ratio on film thickness for the 22 data points is shown in figure 3.6. It is most significant that the computed value of \tilde{A} is approximately unity, since this ensures that the minimum film thickness approaches zero as radius ratio goes to zero. Contour plots for three radius ratios (i.e. α of 16, 9, and 4) are shown in figure 3.7 to demonstrate the influence of geometry. Since the isobars in each case are evenly spaced, the pressure gradients can be easily depicted. As the radius ratio increases, the steeper pressure gradients are predominantly along the rolling direction. This implies that the amount of side leakage decreases as radius ratio increases. Figure 3.8 shows the geometry effects in four different regimes of lubrication, i.e. hard EHL (Hamrock and Dowson (1977a)), soft EHL (Hamrock and Dowson (1978)), piezoviscous-rigid, and isoviscous-rigid (Brewer, et al. (1979)), where H_0 is the minimum film thickness for rectangular contacts. It is assumed that when $\alpha = 150$, the rectangular contact limit is realized. The ratio $H_{min}/H_{min,r}$ approaches the limiting value very quickly in hard EHL, and most slowly in isoviscous-rigid case.

Influence of Materials Properties

A study of the influence of the dimensionless materials parameter G on minimum film thickness has to be approached with caution since in practice it is not possible to change the physical properties of the materials, and hence the value of G , without influencing the other dimensionless parameters considered earlier. Equations (3.9), (3.10), and (3.11) show that when either the materials of the solids (as expressed in E') or the lubricants (as expressed in η_0 and $p_{iv,as}$) are varied, not only does the materials parameter G change, but so do the dimensionless speed U and load W parameters. Only the radius ratio, minimum film thickness and lubricant entrainment direction can be held fixed; and for all results presented in table 3.4, $H_0 = 4.8 \times 10^{-6}$ or $= 1.4 \times 10^{-6}$, $\alpha = 16$, and $\theta = 0$ are adopted.

The results obtained from calculations performed for six values of the dimensionless materials parameter are summarized in table 3.4. The general form of these results, showing how the minimum film thickness is a function of dimensionless materials parameter, is

$$\tilde{C} = C_7 G^{C_8} \quad (3.22)$$

where

$$\tilde{C} = H_0 / [(1 - e^{-0.0387\alpha}) U^{1.266} W^{-0.880}]$$

By applying a least-square power fit to the six pairs of data, the values of C_7 and C_8 are found to be $C_7 = 175.04$ and

$C_8 = 0.386096 \sim 0.386$. The effect of the dimensionless materials parameter on the minimum film thickness is approximated with adequate accuracy as

$$H_0 \propto G^{0.386} \quad (3.23)$$

The variation of dimensionless materials parameter G with dimensionless film thickness H is shown in figure 3.9. The six data points given in table 3.4 are shown in this figure. It is observed that on log-log scale a linear representation of materials parameter versus film thickness is obtained.

Influence of Lubricant Entrainment Direction

If the velocity of the entrainment lubricant is kept constant, but the component velocity in the x direction u and the component velocity in the y direction v is changed, the lubricant entraining angle is modified as shown in equation (3.13), while the other dimensionless parameters (H_0 , G and α) remain constant. The values at which these dimensionless parameters are held constant in the calculations performed to determine the influence of lubricant entrainment direction are

$$\alpha = 20, \quad H_0 = 5.6 \times 10^{-6}, \quad G = 4522, \quad U = 0.75749 \times 10^{-11}$$

The results obtained from calculation performed for 10 values of the lubrication entrainment direction are summarized in table 3.5. A cosine function was chosen for curve fitting, namely

$$\tilde{T} = C_9 \theta \quad (3.24)$$

where

$$\tilde{\tau} = \cos^{-1} (W^{-0.880} / W_0^{-0.880})$$

W_0 denotes the load capacity at $\theta = 0$.

By applying a least square fit to the 10 pairs of data, the values of C_g was found to be $C_g = 0.0200027 \approx 0.0200$. Therefore, the effect of the lubricant entrainment direction on the minimum film thickness can be written as

$$H_0 \propto \cos(1.146 \theta) \quad (3.25)$$

This is shown in figure 3.10. It is significant that when the entraining angle is zero the cosine function turns out to be 1. In other words, there is no effect of lubricant entrainment direction on minimum film thickness. Although the influence of lubricant entrainment direction is investigated for $\alpha = 20$ only, it is found that it can be applied to the cases from $\alpha = 16$ to $\alpha = 30$.

Minimum Film Thickness Formula

The proportionality equations (3.16), (3.19), (3.21), and (3.23) have established how the minimum film thickness varied with the dimensionless load, speed, radius ratio and materials parameter respectively. This enables a composite dimensionless minimum film thickness formula for fully flooded rigid elliptic contact solid lubricated by piezoviscous fluid to be modeled by

$$H = 178 G^{0.386} U^{1.266} W^{-0.880} (1 - e^{-0.0387\alpha}) \quad (3.26)$$

with 85.6 percent confidence (except the small value of α) of ± 8.92 percent of the value from the numerical analysis. Equation (3.26) can be written in dimensional form as

$$h = 178(1/p_{iv,as})^{0.386} (u\eta_0)^{1.266} W^{-0.880} R_x^{1.494} \times (1 - e^{-0.0387 R_y/R_x}) \quad (3.27)$$

Note that the effective elastic modulus E' does not appear in this equation. Case (1) to case (41) in table 3.6 gives the forty-one different cases used in obtaining equation (3.26). To gain more confidence in the application of this empirical equation, some other values of dimensionless U and minimum film thickness H_0 not in the range for curve fitting were checked. The results are shown in case (52) and case (53) of table 3.6. The good agreement between the prediction from the minimum film thickness formula and computed results is encouraging.

4.2 Discussion

A numerical solution for the piezoviscous effect in nonconformal rigid contacts lubricated hydrodynamically has been developed. The following remarks can be made:

(1) It was found that the effective elastic modulus E' does not appear in the dimensional film thickness equation. This corresponds to a rigid contact; there is no effect of elastic properties.

(2) In all cases it was found that if the maximum dimensionless viscosity is less than three, the load capacity is no more than 12 percent larger than the load capacity without

piezoviscous effect. This indicates a very small piezoviscous effects. Here, all the data sets used to generate equation (3.26) have the maximum dimensionless viscosity larger than three.

(3) The exponents of dimensionless load capacity and dimensionless velocity in the minimum film thickness equation for the piezoviscous-rigid regime are between those for piezoviscous-elastic regime and isoviscous-rigid regime.

(4) The film thickness formula according to the side-leakage factor proposed by Hamrock and Dowson (1978) is

$$H = 1.66(GU)^{2/3} [1 - \exp(-0.68*(1.03*\alpha^{0.66}))] \quad (3.28)$$

The more recent formula from the numerical solution by Dowson et al. (1983) is

$$H_0 = 1.66(GU)^{2/3} \left(\frac{\alpha}{1 + \alpha} \right)^{0.6} \quad (3.29)$$

Figure 3.11 compares the minimum film thickness as obtained from equation (3.28) and (3.29) with the present result for $W = 0.6*10^{-6}$ or $0.9*10^{-6}$ and $\alpha = 8$ or 16 . It can be seen that the deviations are large for small values of GU . The deviations result because the load parameter W is not included in equations (3.28) or (3.29), giving erroneous results when the piezoviscous effects are small, i.e. GU small, and because the Barus formula is used for the pressure-viscosity correlation in equations (3.28) and (3.29) cause an overestimate of the minimum film thickness.

(5) The film thickness formula proposed by Houpert (1984) by means of curve fitting of the numerical solution of Dalmaz is

$$H_{pVR} = C H_{IVR}$$

$$C = \exp(0.265A)$$

$$H_{IVR} = \left[\frac{\left(1 + \frac{2}{3\alpha}\right) \frac{W}{U}}{\left(0.131 \tan^{-1}\left(\frac{\alpha}{2}\right) + 1.683\right) \sqrt{128\alpha}} + 2.6511 \right]^{-2}$$

$$A = 12GU \frac{0.177\alpha}{\alpha + 0.778} H_{IVR}^{-1.5} \quad (3.30)$$

A comparison of 20 of the 41 cases used to get the present formula (equation (3.26)) is made with the formula proposed by Houpert. Table 3.7 shows the ratio of H_0 (equation (3.26)) to H_0 (equation (3.30)). The deviation is between 28% to 54%. A possible cause of this difference may be due to Houpert's results imply an incompressible fluid and a Barus pressure-viscosity formula.

3.4 Summary

A procedure for the numerical solution of the complete piezoviscous-rigid regime of lubrication for nonconformal contacts has been demonstrated. In the numerical analysis of the Reynolds equation the parameter $\phi = PH^{3/2}$ was introduced in order to help the relaxation process. A variable-mesh nodal structure was used to provide close spacing in and around the pressure peak. A more realistic formula is used for the pressure viscosity relationship. Lubricant compressibility was also considered.

By using the procedures outlined in the analysis, the influence of the dimensionless load W , speed U , radius ratio α , materials G and lubricant entraining direction θ on minimum

film thickness were investigated. Forty-one cases were used to generate the minimum film thickness relationship

$$H_0 = 178G^{0.386} U^{1.266} W^{-0.880} (1 - e^{-0.0387a})$$

The most dominant exponent occurred in association with the speed parameter; the exponent on the load parameter -0.880 was in between the -2 for isoviscous-rigid regime and -0.073 for piezoviscous-elastic regime. The materials parameter also carries a significant exponent, although the range of the parameter in engineering is limited. The geometry effect shows the same curve as in elastohydrodynamical lubrication, but approaches the limiting value much more slowly. The effect of the direction of lubrication entrainment is a cosine function for the geometries studied. The dimensionless load parameter values used in obtaining the preceding equation are varied over a range of one order of magnitude. The dimensionless speed values are varied over 5.6 times the lowest value. The radius ratio is varied from 0.2 to 64 (a configuration approaching a line contact). Conditions corresponding to the use of solid materials of steel, bronze and silicon nitride and lubricants of paraffinic and naphthenic mineral oils are considered in obtaining the exponent in the dimensionless materials parameter. Contour plots are shown that indicate in detail the pressure developed between the solids.

Chapter 4

ELASTIC DEFORMATION OF ELLIPTICAL CONTACTS

In solving various engineering problems, such as the point contact EHL problems and other frictionless contact problems, the elastic surface deformation is often determined by equation (2.9). However, there are two important problems associated with the integration of equation (2.9) as far as numerical methods are concerned. The first problem arises from the singularity at $x = x'$ and $y = y'$. The second is that the amount of work in the numerical integration is considerably large. This is due to the fact that the whole area must be integrated in order to evaluate the deformation at a certain point and that in the repeated calculation, such as in the numerical solution of the EHL problem by means of iteration, the deformation of every node on the finite difference grid must be evaluated for each iteration. The method of dealing with the singularity is to use a polynomial function to approximate the pressure distribution so that the analytical solution for the integration on the grid element can be obtained. In order to reduce the amount of the calculation an influence coefficient matrix D is introduced, in which the element D_{ij}^{kl} is defined as the deformation occurring at node (k,l) when a unit of pressure had been applied to node (i,j) . Consequently equation (2.9) can be described by the following discrete form:

$$d_{kl} = \frac{2}{\pi} \sum_{i=1}^I \sum_{j=1}^J D_{ij}^{kl} p_{ij} \quad (4.1)$$

All the values of D_{ij}^k must be completely computed only once, and the deformation can be calculated by only applying the expression (4.1).

These approaches were first successfully developed by Hamrock and Dowson (1974) and applied to the EHL problems. They assume that the pressure on each element can be replaced by a constant value, that is the whole pressure distribution is replaced by blocks of uniform pressure. In this way, an analytical expression for the integration of the deformation is worked out and the deformation of every node expressed as a linear combination of the nodal pressures. In the solution of Ranger, Ettles, and Cameron (1975), a bilinear interpolating function is used to approximate the practical pressure distribution. Evans and Snidle (1981, 1982) employed a different method which was first presented by Biswas and Snidle (1977). For grid elements without singularity, they adopted directly Simpson's rule for integration, and for those with singularity they used a biquadratic polynomial function to express the pressure distribution approximately. In this way, an analytical solution for the integration is developed without direct expression for the deformation as a linear combination of the nodal pressures. Following in these footsteps, Hou Keping, Zhu Dong, and Wen Shizhu (1984) employed the biquadratic polynomial function for approaching the pressure distribution on all grid elements. An influence coefficient matrix is introduced to reduce the amount of calculating work when repeated

calculations of the elastic deformation is needed. A comparison of computing methods by Ranger et al., Biswas and et al., and the authors was presented. It is found that the method proposed by Hou Keping et al. has higher numerical accuracy and uses less computational time than the others. The reason that Hou Keping's method is more accurate than Biswas's is because of the fact that the integrand of the deformation integral varies very rapidly around the region of singularity and this can lead to significant errors in evaluating the contribution to the total deformation by the area surrounding the singularity. However, for a $(2n+1)*(2m+1)$ finite difference grid, the influence matrix introduced by Hou Keping et al. is composed of $(n*m)*(2n*2m*9)$ elements, which is not welcomed even in today's powerful computer. It means that only a relatively coarse finite difference grid can be used in the EHL problems.

In the present study, a biquadratic polynomial expressed in Lagrange form is used to approximate the pressure distribution on all grid element. An influence matrix whose coefficient is dependent on the geometric factors and the distance between node (k,l) and node (i,j) only is introduced to express the deformation of every node as a linear combination of the nodal pressures. In this way, only $(2n*2m*9)$ elements is needed in the influence matrix for a $(2n+1)*(2m+1)$ equidistant rectangular grid. The computational time and computer storage size for the influence coefficient matrix is tremendously reduced. For example, for a

(21*21) equidistant rectangular grid ($m=n=10$) the elements of influence matrix is reduced from 36 0000 to 3600. Incorporating this new elasticity approach with reasonable finite difference grid into the EHL problems is now applicable.

4.1 The Form of Analytical Solution

On a nine-node element (e) of any nonequidistant rectangular grid, a biquadratic polynomial is used to approach the pressure function (fig. 4.1)

$$p(x,y) = \sum_{n=1}^g \psi_n^e(x,y) p_n^e \quad (4.2)$$

where p_n^e stand for pressure at the nine nodal points, $\psi(x,y)$ are nine biquadratic functions which can be expressed as the following

$$\begin{aligned} \psi(x,y) &= \prod_{\substack{r=1-1 \\ r \neq m}}^{1+1} \left(\frac{x - x_m}{x_r - x_m} \right) \prod_{\substack{s=j-1 \\ s \neq n}}^{j+1} \left(\frac{y - y_n}{y_s - y_n} \right) \\ &= a_1 (x^2 + a_2 x + a_3) (y^2 + a_4 y + a_5) \\ m &= i-1, i, i+1 \\ n &= j-1, j, j+1 \end{aligned} \quad (4.3)$$

where $a_1 - a_5$ only depend on the grid geometry. The deformation at node (k,l) caused by the pressure distribution on the element (e) is:

$$\begin{aligned} d_{kl}^e &= \frac{2}{\pi} \sum_{n=1}^g p_n^e I_n^e \\ I_n^e &= \iint_e \frac{\psi_n^e(x,y)}{\sqrt{x^2 + y^2}} dx dy \end{aligned} \quad (4.4)$$

After evaluating I for all elements, the total deformation results:

$$a_{kl} = \frac{2}{\pi} \sum_e \sum_n p_n^e I_n^e \quad (4.5)$$

By using the nine values I_n^e of every element, the total deformation matrix on the whole region can be composed. If a function $FUN(x,y)$ is defined as follows,

$$\begin{aligned} FUN(x,y) = \frac{a_1}{120} \bigg\{ & y \ln(t+x) [120 a_3 a_5 + 60 y a_3 a_4 - 20 y^2 (a_5 - 2a_3) \\ & - 15 y^3 b_4 - 12 y^4] + x \ln(t+y) [120 a_3 a_5 + 60 x a_2 a_5 \\ & + 20 x^2 (2 a_5 - a_3) - 15 x^3 b_2 - 12 x^4] + t [60 x a_3 a_4 \\ & + 60 y a_2 a_5 + xy (20 a_5 + 20 a_3 - 15 x a_2 - 15 y a_4) \\ & + t^2 (40 a_2 a_4 + 30 y a_2 + 30 x a_4 + 12 xy)] \bigg\} \quad (4.6) \end{aligned}$$

where

$$t = \sqrt{x^2 + y^2}$$

the deformation integral becomes

$$I = [FUN(x_1, y_1) + FUN(x_3, y_3) - FUN(x_1, y_3) - FUN(x_3, y_1)]$$

Since

$$FUN(x, -y) = -FUN(x, y) \text{ and } FUN(-x, y) = -FUN(x, y)$$

it can be proved that $I_n^e(x, y) = I_n^e(x, -y)$ and

$I_n^e(x, y) = I_n^e(-x, y)$. It means that I is an even

function with respect to x and y . Therefore the coefficient of the influence matrix depends on the geometric factors of the grid and the distance between the central of the grid point and the

deformation point only. The size of the influence coefficient matrix is thereby tremendously reduced.

4.2 Discussion

Hou Keping et al. have demonstrated the advantage of employing the biquadratic interpolating polynomial for approaching the pressure distribution on all grid elements by comparing the numerical accuracy with the methods proposed by Ranger et al. and Biswas et al. The comparison between the present approach and Hamrock's approach is shown in table 4.1. The results of table 4.1 show that for a given mesh the best accuracy in the calculation of d_{\max} and Δh_{\max} is obtained by the present approach. The accuracy of Δh_{\max} is also shown since it is a measure of the flatness of the film shape under the Hertzian pressure distribution. This aspect is extremely important at high load, where the elastic deformations are two or three orders of magnitude larger than the film thickness.

4.3 Concluding Remarks

In conclusion, the calculation procedure presented here has the advantages of higher numerical accuracy, less computing time and smaller storage size required for the influence matrix and is easier to apply than other methods proposed to date.

Chapter 5

THE EFFECT OF SURFACE ROUGHNESS ON ELASTOHYDRODYNAMIC
LUBRICATION OF POINT CONTACT

Detailed studies of surfaces and the quantification of surface features of importance in tribology are recent, mid-twentieth century developments. An understanding of the effect of surface roughness on lubrication is important in specifying engineering surface finishes, analyzing bearing failures, and predicting the mode of lubrication in a practical situation and the breakdown of full film lubrication. The first attempts to include roughness effect considered deterministic striations such as sinusoidal waves oriented parallel or perpendicular to the direction of motion of the bearing components, i.e. longitudinal and transverse roughness (Elrod, 1977). The real bearing surfaces, however, have a more or less random distribution of roughness. The study of surface roughness effects in lubrication has gained increasing attention with the introduction of stochastic concepts by Tzeng and Saibel (1967). Since then stochastic process theory has been utilized on various bearing performances. Tzeng and Saibel (1967) utilized the stochastic approach for a slider bearing with one-dimensional transverse roughness. Christensen and Tonder (1971, 1972, 1973) developed the stochastic Reynolds equation for transverse and longitudinal roughness and use this equation to analyze the hydrodynamic lubrication of slider and journal bearings. These approaches

involve deriving Reynolds type equations which govern the mean pressure in rough bearings, and these equations have been utilized by many workers to predict rough bearing performance. However, these stochastic Reynolds equations are restricted to two types of roughness structures: one-dimensional ridges oriented either transversely or longitudinally. These are the limiting cases of roughness found on real surfaces. The treatment of general random roughness is a recent development. Two approaches have been developed. The first, due to Bush et al. (1982), introduces roughness as a perturbation to the smooth bearing and averages over an ensemble of roughness configurations. The perturbation method is inherently prevented from being applied to bearings in which contacts occurs since the analysis relies on small-amplitude roughness. However, this assumption allows the effects of roughness to be determined from linear equations whereas the basic Reynolds equation is nonlinear. Roughness effects are therefore relatively easy to calculate. The second method is the flow factor model introduced by Patir and Cheng (1978). In their formulation, the actual flow between rough surfaces is equated to an averaged flow between nominally smooth surfaces, while parameters describing the roughness are included in the Reynolds equation through the flow factors, which are obtained by numerical flow simulation or the height readings of a real surface. Thus any type of roughness structure can be analyzed. Another

advantage of this method is that it can be extended to the mixed lubrication regime in which contact between bearing surfaces occurs.

Using the flow factor formulation, Patir and Cheng (1979) performed a calculation on the inlet half of a elastohydrodynamic line contact with the Ertel-like approximation for the inlet shape as described in Chapter 1. Recently, Tripp and Hamrock (1984) did the complete numerical solution of the isothermal elastohydrodynamic lubrication of line contact on rough surfaces with the same flow factor approach.

In the present study the flow factor model developed by Patir and Cheng (1978) is adopted to study the surface roughness effect on the elastohydrodynamic lubrication of point contact, which essentially takes into account the side leakage effect of nonconformal contacts. Conditions typical of an EHL contact in the piezoviscous-elastic regime with a developed pressure spike near the outlet are investigated. The Roelands formula is used for the pressure-viscosity relationship. The lubricant has been assumed to be compressible and the system to be isothermal. Results are presented demonstrating the effect of surface roughness on the film shape, pressure distribution, and coefficient of rolling friction in a ball bearing.

5.1 Background

The general background for the surface roughness and the stochastic aspect of flow will be described in this section. Tensor notation is used for simplicity.

Surface Roughness

The significance of advanced surface research is becoming apparent where surface systems play an important role affecting the behavior of a physical process. The problems in design, such as lubrication, stiffness of joints, strength of parts, fatigue, accuracy of dimensional chains and kinematic trains, dynamics of machine tools, etc., cannot be satisfactorily approached without information about surface configurations that exhibit a profound random character.

In general terms, the roughness of a solid is defined as any departure of the actual surface height from the ideal datum level, also known as nominal level, occurring as a result of the physical and chemical methods used to prepare the final surface as well as microstructure of the material (fig. 5.1).

To describe the rough surfaces in a statistical sense requires knowledge of the height distribution and the correlation function for these heights. These depend upon the scale selected for measurement of surface features and are not intrinsic properties of the surface. The height distribution is the probability that a single point in the surface selected at random lies at or below a given height, while the correlation functions

describe how the probability of finding one point at a given height depends on the heights of the surrounding points. This is best illustrated by the two point correlation function which is the ensemble-averaged product of heights at two points selected at random with fixed spacing between them. For a general surface, correlation depends on direction as well as magnitude of the displacement of the two points and is determined by the typical dimensions of surface features in that direction.

Reynolds Roughness and Stokes Roughness

When Reynolds equation is used to study the surface roughness effect on lubrication, the roughness should be the Reynolds type which requires long wavelength and small amplitude for the asperities relative to film thickness, and is labeled Reynolds roughness (Elrod, 1978). If the wavelength is so small as to necessitate the application of the Navier-Stokes equation, it is referred to as Stokes roughness. Only the Reynolds roughness will be considered in this study.

Statistical Aspects of Flow

The primitive parameters of the problem are given in figure 5.1, which shows the x-direction of a lubricant film moving between two rough solid boundaries at heights z_a and z_b . The translational velocities \vec{u}_a and \vec{u}_b of these boundaries lie in the (x,y) plane and are independent of position and time. Thus, there is no rigid-body rotation and no transient effect. The actual surface height $z_{a,b}$ is considered to be simply the

sum of the specified nominal height and a random roughness height $\delta_{a,b}$. The random variable $\delta_{a,b}$ is a stationary stochastic process distributed with standard deviation (rms value) $\sigma_{a,b}$ about zero mean. We need also the standard deviation of the roughness combination $(\delta_b - \delta_a)$, which in cases where cross-correlation is absent will be just $(\sigma_a^2 + \sigma_b^2)^{1/2}$. For convenience the boundary variables with the following combinations are introduced:

film thickness	mean surface height	
$h = z_b - z_a$	$z = \frac{1}{2} (z_a + z_b)$	
slip velocity	mean surface velocity	(5.1)
$\vec{v} = \vec{u}_b - \vec{u}_a$	$\vec{u} = \frac{1}{2} (\vec{u}_a + \vec{u}_b)$	

The volume flow Q_1 per unit width due to the pressure gradient $\partial_1 p$ and the entraining velocity u_1 , under the assumptions of the Reynolds equation described in Chapter 2, is given by

$$Q_1 = -\frac{h^3}{12\eta} \partial_1 p + hu_1 \quad (5.2)$$

The continuity equation for incompressible flow $\partial_1 Q_1 = -\partial h/\partial t$ is applied to derive the Reynolds equation for this problem:

$$\partial_1 \left(\frac{h^3}{12\eta} \partial_1 p \right) = -v_1 \partial z_1 \quad (5.3)$$

where the Euler transform has been used to interchange time and space derivative of h .

The central idea for the flow factor method is to recast the flow vector Q_i in terms of the averaged pressure and film thickness, p^* and h^* replacing their true fluctuating values p and h . To achieve this, the effects of roughness must be included explicitly. The form introduced by Patir and Cheng for incompressible laminar flow is:

$$Q_i = - \frac{h^{*3}}{12\eta} \phi_{ij}^p \partial_j p^* + h^* U_i - \frac{\sigma}{2} \phi_{ij}^s v_j \quad (5.4)$$

where tensors ϕ_{ij}^p and ϕ_{ij}^s are termed, respectively, the pressure (or Poiseuille) and shear flow factors. The shear flow factor ϕ_{ij}^s accounts for flow produced by roughness in the presence of slip even when the pressure gradient vanishes. ϕ_{ij}^p can be thought as a correction factor, comparing the mean pressure flow in rough bearings to that of smooth bearings having the same nominal geometry, namely additional entrainment due to roughness under pure rolling conditions. Fluctuations in flow are now fully incorporated into the flow factors. It is central to the method to assume these fluctuations are negligible compared to those of p and h . The continuity condition $\partial_i Q_i = -\partial h/\partial t$ may now be applied to Q_i^* instead of Q_i yielding

$$\partial_i \left(\phi_{ij}^p \frac{h^{*3}}{12\eta} \partial_j p^* \right) = - v_j \partial_i \left(z \delta_{ij} + \frac{1}{2} \sigma \phi_{ij}^{s*} \right) \quad (5.5)$$

This is the modified Reynolds equation for surfaces in translational motion. At this stage, Patir and Cheng adopted the direct approach of numerical simulation of the roughness surfaces

to obtain the flow factors. Tripp (1983) extended the flow factor model proposed by Patir and Cheng (1978) and expressed the flow factors on the roughness parameters in simple closed form by analytical method. In his approach, the pressure term is expanded as a perturbation series by regular perturbation approximation. Then green function technique is applied to solve the terms in the series. It is found that to second order the flow factors are determined by just two parameters: the rms surface height referenced to h , which in the usual notation becomes the film parameter $\Lambda = h/\sigma$, and the anisotropy of typical surface asperities given by γ the ratio of the two-point correlation lengths parallel and perpendicular to the lay direction of the surface texture. Purely transverse, isotropic, and purely longitudinal roughness structures correspond to $\gamma = 0, 1, \infty$, respectively. Surfaces with $\gamma > 1$ are longitudinally oriented (fig. 5.2). In this study, the flow factors express as a function of the film parameter Λ and the anisotropy index γ proposed by Tripp (1983) are used.

5.2 Method of Calculation

For an incompressible lubricant, the modified Reynolds equation for surfaces in translational motion is introduced in last section for generality. For compressible lubricant, the modified Reynolds equation for rotational motion about fixed axes has slight differences from equation (5.5) since now the density is included in the flow vector and the term $\partial(\rho h)/\partial t = 0$ in the

continuity equation is zero. The corresponding form of equation (5.5) becomes

$$\partial_i \left(\phi_{ij}^p \frac{\rho h^3}{12\eta} \partial_j p^* \right) = u_i \partial_i (\rho h) - \frac{1}{2} \sigma v_j \partial_i \rho \phi_{ij}^s \quad (5.6)$$

The flow factors are locally deterministic and a second order perturbation calculation using Gaussian forms both for the height distribution and the two-point autocorrelation function (Tripp, 1983) yields the result

$$\left. \begin{aligned} \phi_{xx}^p(\Lambda, \gamma) &= 1 + 3 \frac{\gamma - 2}{\gamma + 1} \Lambda^{-2} \\ \phi_{yy}^p(\Lambda, \gamma) &= \phi_{xx}^p(\Lambda, \frac{1}{\gamma}) \\ \phi_{xx}^s(\Lambda, \gamma) &= \frac{3}{\gamma + 1} \Lambda^{-1} \\ \phi_{yy}^s(\Lambda, \gamma) &= \phi_{xx}^s(\Lambda, \frac{1}{\gamma}) \end{aligned} \right\} \quad (5.7)$$

with all other components zero. In this representation the x and y directions coincide with the roughness axes determined by the surface lay, from which the general case is readily obtained by coordinate rotation. The forms given by equation (5.7) are compared with the results in Patir and Cheng (1978, 1979) to those computed directly from an ensemble of generated rough surfaces with generally good agreements shown by figures 5.3 and 5.4 in the physically realistic range of Λ .

Choosing the pure rolling condition for illustrative purposes, the flow vector of equation (5.4) in explicit notation is

$$\left. \begin{aligned} Q_x &= -\frac{h^{*3}}{12\eta} \phi_{xx}^* p \frac{\partial p^*}{\partial x} + u_x h^* \\ Q_y &= -\frac{h^{*3}}{12\eta} \phi_{yy}^* p \frac{\partial p^*}{\partial y} + u_y h^* \end{aligned} \right\} \quad (5.8)$$

Applying the continuity equation to equation (5.8) leads by an Euler transform to the Reynolds equation appropriate to the case illustrated in this study:

$$\begin{aligned} \frac{\partial}{\partial x} \left(\phi_{xx}^* \frac{\rho h^{*3}}{\eta} \frac{\partial p^*}{\partial x} \right) + \frac{\partial}{\partial y} \left(\phi_{yy}^* \frac{\rho h^{*3}}{\eta} \frac{\partial p^*}{\partial y} \right) \\ = 12U \left[\cos \theta \frac{\partial(\rho h^*)}{\partial x} + \sin \theta \frac{\partial(\rho h^*)}{\partial y} \right] \end{aligned} \quad (5.9)$$

The following dimensionless groups are introduced for nondimensionalization

$$\bar{x} = \frac{x}{b}, \quad \bar{y} = \frac{y}{a}, \quad \bar{\rho} = \frac{\rho}{\rho_0}, \quad \bar{\eta} = \frac{\eta}{\eta_0}, \quad H^* = \frac{h^*}{R_x}, \quad p^* = \frac{p}{E'} \quad (5.10)$$

where a and b are semimajor and semiminor axis of contact ellipse.

By substituting these dimensionless groups into the governing equations (5.9), (2.6), (2.7), and (2.14), the dimensionless equations describing the piezoviscous-elastic lubrication on rough bearings is obtained.

The modified Reynolds equation is

$$\frac{\partial}{\partial \bar{x}} \left(\phi_{xx}^* \frac{\bar{\rho} H^{*3}}{\bar{\eta}} \frac{\partial p^*}{\partial \bar{x}} \right) + \frac{\partial}{\partial \bar{y}} \left(\phi_{yy}^* \frac{\bar{\rho} H^{*3}}{\bar{\eta}} \frac{\partial p^*}{\partial \bar{y}} \right) = 12U \left[\cos \phi \frac{\partial(\bar{\rho} H)}{\partial \bar{x}} + \sin \theta \frac{\partial(\bar{\rho} H)}{\partial \bar{y}} \right] \quad (5.11)$$

where $U = \eta_0 U/E'R_x$

The pressure-viscosity formula from Roelands (1966) is

$$\bar{\eta} = \left(\frac{\eta_{\infty}}{\eta_0} \right) [1 - (1 + p^* E'/c)^2] \quad (5.12)$$

where $\eta_{\infty} = 6.31 \times 10^{-5}$ Ns/m and $c = 1.96 \times 10^8$ Ns/m².

The pressure-density formula from Dowson and Higginson (1966) is

$$\bar{\rho} = 1 + \frac{s p^* E'}{1 + t p^* E'} \quad (5.13)$$

where s and t are constants that depend on the fluid.

The film thickness formula is

$$H^* = H_0^* + \frac{s(\bar{X}, \bar{Y})}{R_x} + \frac{d(\bar{X}, \bar{Y})}{R_x} \quad (5.14)$$

Boundary Conditions

The boundary conditions are similar to what have been described in chapter 2 for piezoviscous-rigid lubrication.

(a) At the cavitation boundary,

$$p^* = \frac{dp^*}{d\bar{x}} = \frac{dp^*}{d\bar{y}} = 0$$

This condition is commonly known as the Reynolds condition.

(b) At the edges of the computation zone the pressure is zero. The fully flooded conjunction is considered.

Numerical Analysis

The modified Reynolds equation (5.11) for P is transformed into an equation for $\Phi = P^* H^{*3/2}$ and discretized by central

finite difference expansion on a uniform grid scaled to the Hertzian contact ellipse. This scale is chosen instead of the effective radius R_x introduced in Chapter 2 for piezoviscous-rigid lubrication because it's found that it can speed up the convergence process when the elastic deformation is taken into account. Thus a load capacity has to be specified in the beginning to determine the major and minor axes of the Hertzian ellipse instead of giving a specific H_0 to get a certain load capacity. It means that an extra outer loop has to be satisfied in addition to the convergence of pressure distribution through the method of frozen coefficients. Hence, the nominal dimensionless separation of the rigid solids H has to be adjusted to compensate for the difference between the actual load obtained by integration of the convergent pressure distribution through the method of frozen coefficients and the initially assumed load. A flow chart is shown in figure 5.5. A damping factor is always necessary for each iteration through the method of frozen coefficients to get a convergent pressure distribution. Extensive computing time is required since the elastic deformation is involved in the film thickness formula. The numerical scheme described in Chapter 4 is adopted for the calculation of elastic deformation.

Coefficient of Friction

The shear force per unit length acting on solid a can be written as

$$f_a = \int \left(\eta \frac{du}{dz} \right)_{z=z_a} dx \quad (5.15)$$

Integrating the reduced Navier-Stokes equation for dp/dx across the film gives

$$\eta \frac{du}{dz} = \frac{[2(z - z_a) - h]}{2} \frac{dp}{dx} - \frac{\eta(u_a - u_b)}{h} \quad (5.16)$$

Substituting this equation into equation (5.15) gives

$$f_a = - \int \left[\frac{h}{2} \frac{dp}{dx} + \frac{\eta(u_a - u_b)}{h} \right] dx \quad (5.17)$$

The shearing force per unit length acting on solid b can be written as

$$f_b = \int \left(\eta \frac{du}{dz} \right)_{z=z_b} dx \quad (5.18)$$

Making use of equation (5.16) gives equation (5.18) as

$$f_b = \int \left[\frac{h}{2} \frac{dp}{dx} - \frac{\eta(u_a - u_b)}{h} \right] dx \quad (5.19)$$

The normal pressure p acting on the two solids also has a net x-component $w_x = w_{ax} + w_{bx}$ which can be expressed by

$$w_x = \int h \frac{dp}{dx} dx \quad (5.20)$$

The shear and pressure forces thus balance, satisfying the equilibrium condition

$$f_a - f_b + w_x = 0 \quad (5.21)$$

For a pure rolling condition, we have

$$f_a = -f_a = -\frac{1}{2} \int h \frac{dp}{dx} dx \quad (5.22)$$

in which the explicit appearance of n in equations (5.15) and (5.18) is now hidden. The coefficient of rolling friction for either solid a or b is given by

$$\mu = |f^*|/\mu \quad (5.23)$$

where the expectation value of the shear force is given in second order perturbation theory by

$$f^* = -\frac{1}{2} \int h^* \frac{dp^*}{dx} \left(1 + \frac{3}{\gamma + 1} \Lambda^{-2}\right) dx \quad (5.24)$$

5.3 Results and Discussion

The results for a steel bearing in a pure rolling condition is presented. The values of the parameters are shown in table 5.1. Equation (5.7) for the pressure flow factor along the x direction shows a crossover from impeded to enhanced flow along the rolling direction as γ increases through the critical value 2. Representative of asperities on either side of this crossover we have examined the isotropic case, $\gamma = 1$ and the case where asperity are longer in the flow direction by a factor $\gamma = 3$. The range of the film parameter Λ_{mo} studied is from 2 to 10, where $\Lambda_{mo} = h_{mo}/\sigma$, h_{mo} demotes the minimum film thickness for the contact operation with smooth surfaces.

Filmshape Effects

Values of the minimum and central film thickness as a function of Λ_{mo} for $\gamma = 1$ and $\gamma = 3$ are shown in figures 5.6 and 5.7. Values are normalized to the smooth value for comparison. (The same normalization procedure will be followed in studying the variation of pressure distribution and coefficient of friction. The smooth curves joining the computed points have been sketched simply to aid visualization.) For $\gamma = 1$ the minimum and central film thickness grows as Λ_{mo} becomes smaller, while for $\gamma = 3$ minimum and central film thickness decreases as Λ_{mo} becomes smaller. This trend is consistent with the results obtained by Tripp and Hamrock (1984) on line contact of incompressible lubricant but more pronounced. However, this significant filmshape effect agrees with their recent results (Tripp and Hamrock, 1985) on line contact of compressible lubricant by incorporating the improved elasticity calculation into the EHL numerical scheme. It is also noted that isotropic or transverse asperity has stronger effect than longitudinal asperity for the point contact case.

Pressure Effect

The variation of pressure spike under different values γ of Λ and is provided in figure 5.8. For $\gamma = 3$ the pressure spike grows as Λ_{mo} becomes smaller, while for $\gamma = 1$ pressure spike decreases as Λ_{mo} becomes smaller. The work of Tripp and Hamrock (1984) didn't observe any significant change in pressure

spike. A more recent work (Tripp and Hamrock, 1985) indicates that there are small variation in the line contact case of incompressible lubricant and slightly more variation in the line contact case of compressible lubricant.

Rolling Friction Effects

Results of the coefficient of rolling friction calculations based on equation (5.24) are plotted in figure 5.9. For $\gamma = 3$, rolling friction grows as Λ_{mo} becomes smaller, while for $\gamma = 1$, rolling friction grows as Λ_{mo} becomes larger. The longitudinal texture $\gamma = 3$ produces smaller change in μ . The result of Tripp and Hamrock (1984) on the line contact of incompressible lubricant indicate that rolling friction grows as becomes smaller for both $\gamma = 1$ and 3, longitudinal texture showing the larger effect. However, in their recent work on line contact of the compressible lubricant, it was found that rolling friction increases for isotropic or transverse roughness, but decreases for longitudinal roughness $\gamma > 2$, and the friction effect grows as Λ_{mo} becomes smaller. This trend is consistent with the results presented here.

5.4 Concluding Remarks

The flow factor model proposed by Patir and Cheng is adopted to study surface roughness effects on elastohydrodynamic lubrication of a point contact. The flow factors are obtained from Tripp's analytical approach.

The numerical method presented in Chapter 3 incorporating the improved elasticity calculation described in Chapter 4 is used to solve the modified Reynolds equation successfully. The system we have investigated is a compressible, isothermal, Newtonian fluid with Roelands viscosity entrained in pure rolling.

Results are obtained for a range of film parameter A_{mo} from 2 to 10. Anisotropy effects are included by studying both isotropic asperities $\gamma = 1$ and asperities three times longer on average in the flow direction than that transverse to it, $\gamma = 3$. The surface roughness effect on film shape, pressure distribution, and rolling friction are presented.

Chapter 6

PARCHED ELASTOHYDRODYNAMIC LUBRICATION

A starved subdivision of EHL was originally developed because the fully flooded numerical solution as applied to instrument ball bearings calculated films which were much too thick. A starvation correction for a reduced oil supply was constructed by assigning an arbitrary upstream location for the beginning of pressure build up (Orcutt and Cheng, 1966; Castle and Dowson, 1972). Calculated films indeed became thinner as the inlet meniscus was brought toward the Hertz area, and the correspondence was confirmed for film thickness measured by interferometry in a ball-on-plane geometry (Wedeven et al., 1971). As a result, most starved elastohydrodynamic calculations presently use some form of inlet meniscus position criterion.

However, the inlet meniscus position starvation, which is a steady state theory, cannot apply to instrument ball bearings. These have no inlet meniscus because they operate without any free bulk oil, and show long term transients in film thickness (Kingsbury, 1984). Since inlet meniscus position starvation has become entrenched in the literature (Hamrock and Dowson, 1981). It seems useful to define a parched elastohydrodynamic lubrication framework which would be able to accommodate at least instrument ball bearings.

Parched lubrication is used in instrument bearings for two reasons: the minimum torque demand and because the spin axis

defined in space by a pair of parched bearings is the most stable of any lubrication regime.

The work presented below was performed with Dr. E. Kingsbury of C.S. Draper Laboratory during his stay at NASA Lewis Research Center as an NRC fellow.

6.1 Parched Transient

A ball bearing is parched if oil films are so thin outside the Hertz contacts that there is no flow under service accelerations. This is the situation in instrument bearings.

As described by Singletary (1966), it is reasonable to say that oil films are immobile outside the Hertz areas in a parched bearing. This means that there is no inflow back into the Hertz track between ball passes in a parched bearing.

The oil under pressure inside a Hertz contact on the other hand is not immobile. It may be very viscous but as rolling continues it is gradually forced sideways out of the track. Since it is not replaced (absent a special oil replacement system), there is a long term transient in film thickness. Such transients in instrument ball bearings have been recognized for many years (Archibald and Blasingame, 1963; Horch, 1963). Neither fully flooded nor starved EHL theory allows for transients. Both calculate film thickness from material properties, geometry, and kinematics, using a steady state formulation; thickness is a time-constant dependent variable. In contrast, a parched EHL film

thickness is given or set as an independent variable at the start of a calculation or experiment.

6.2 Basic Speed Ratio

Basic speed ratio (BSR) is a characteristic number describing the kinematic configuration of an angular contact ball bearing.

It is defined as

$$B = \dot{\phi}/S \quad (6.1)$$

where $\dot{\phi}$ is the ball spin angular velocity and S is the total speed of the bearing, i.e., the algebraic difference in race angular velocities. For perfect ball-race coupling and zero ball centrifugal force B is independent of bearing mode, i.e., inner, outer, counter, or joint rotation. For zero slip and no dynamic effects

$$B = \frac{d_m^2 - d_b^2 \cos^2 \beta}{2 d_b d_m} \quad (6.2)$$

where the bearing geometry is defined by the pitch diameter d_m , the ball diameter d_b and the zero load zero speed contact angle β . The first-order basic speed ratio is a function of geometry only.

The basic speed ratio can be measured extremely accurately in a bearing run in the counter race rotation mode at zero ball orbit rate. For stationary ball centers individual spin rates can easily be obtained to 1 part in 10^5 , and the BSR computed from its definition. Since the first order BSR is a geometric constant, any measured change reflects some deviation from the

first order assumptions. This is useful for example in determining running conditions where specific assumptions break down, and in evaluating numerical modes of bearing performance (Kingsbury, 1984).

In a successful running bearing balls and races are separated by an EHL film. Since ball spin is driven by traction forces through this film, BSR gives a sensitive measurement of ball-lubricant-race coupling (Kingsbury, 1980). For perfect coupling (no lubricant, zero drag slip), BSR has a maximum value set by bearing geometry that decreases with increasing spin drag slip. Coupling, in turn, depends on film rheology and thickness.

6.3 Experimental Consideration

In the experiment to be described here a parched, retainerless 108 size bearing was run under pure axial load in the counter race rotation mode for zero ball orbit rate. No lubricant make up system was used so the oil lost from or modified within the Hertz areas was not replaced. The resulting transient in ball-lubricant-race coupling was followed by measuring the basic speed ratio of the bearing (Kingsbury, 1980).

An initial uniform parched EHL film can be established on bearing balls by taking them from a dilute solution of oil in a volatile solvent. Its thickness can be adjusted by changing concentration and estimated by weighing. The lubricant deposited in this way for these tests is SRG mineral oil.

Figure 6.1 shows the counter rotation fixture which was built for 108 size bearings. Table 6.1 gives measured geometry for the single 108 bearing which was used in all test. All tests were run at a total speed of 62 Hz (3720 rpm) and axial load of 1344 N. The resulting bearing operating parameters are given in table 6.2.

6.4 Observation

Figures 6.2 and 6.3 show transients found in the 108 bearing: basic speed ratio is plotted against time. The independent variable in the tests is the initial film thickness, 2×10^{-7} m for figure 6.2, and 8×10^{-8} m for figure 6.3.

General behavior in the two tests is the same: an initial decrease (not shown in fig. 6.2 because it was over too quickly to get a reading of speed ratio), a constant period, then a linear increase. The differences are an extreme time compression and generally higher levels of speed ratio for the thinner film.

The thin film test was stopped after 436 s because each ball, originally spinning about a fixed, stable axis, had begun to tumble. This is taken to show a significant change in the net surface traction moment applied to the ball through its EHL films (Dormgold and Klaus, 1968), a precursor of penetration and surface damage. No sign of tumbling was seen in the thick-film test, which was suspended after 20 000 s.

6.5 Discussion

The decreasing part of figure 6.2 is not understood. It has been seen in many other tests and is tentatively ascribed to some

average rearranging of the original film coverage, which is not really "uniform" after deposition.

The constant parts of figures 6.2 and 6.3 can be explained assuming they really show a small increasing trend which present measuring techniques for speed ratio are too coarse to resolve. They result as fluid oil pumped sideways out of the Hertz track under repeated high pressure transients. Oil is not replaced since there is no flow in a parched EHL film outside Hertzian contacts. Fluid mechanics (Kingsbury, 1973) or plasticity equations would describe transient thinning of a parched EHL film. In addition to the variable rheologic circumferential slip measured by basic speed ratio, an angular contact bearing always produces constant kinematic pivoting slips (Kingsbury, 1984). As the EHL film thins, shear rates due to this slip increase, concentrating mechanical shear energy into smaller and smaller Hertz volumes. When the concentration is high enough polymerization or oxidation reactions are activated, chemically (irreversibly) solidifying the lubricant. The linear increase in speed ratio (coupling) shown in figures 6.2 and 6.3 are attributed to this process (Kingsbury, 1978). The rate at which degradation reactions occur is proportional to $\exp(-E_h/Ttu)$, where E is an activation energy, T is a limiting shear stress, h is film thickness, u is (pivoting) slip velocity, and t is contact transient time. Reaction rate increases exponentially as thickness decreases, without any film penetration.

In the experiment degradation is detected by washing balls taken from a parched test with the solvent used to deposit virgin oil. The parched oil film is gone, inside and outside the Hertz track, but a light brown insoluble polymeric film remains. Diffuse light photographs then show the amount and placement of degradation product. The polymer is gone after wiping with a soft cloth, and there is no evidence of surface damage. The polymer was unpenetrated during running. This is one justification for separation of the parched and mixed EHL regimes.

6.6 Summary and Conclusions

Elastohydrodynamic lubrication does not go directly from starved to mixed, at least in instrument ball bearings. The intervening parched regime is characterized by zero lubricant flow outside the Hertz area, slow crossflow inside the Hertz area, and long term transients in film thickness. Parched EHL treats initial film thickness as an independent variable, in contrast to classical EHL which calculates thickness from other variables. Failure of a parched film is by oxidation or polymerization rather than by asperity penetration. The degradation reactions are activated by mechanical shear. Infinite steady state parched operation of a bearing therefore requires an oil makeup system with inflow adjusted to crossflow. Parched operation requires the least driving torque and gives the best spin axis definition possible in a given bearing.

CHAPTER 7

SUMMARY

The lubrication of nonconformal contacts is characterized by poor conformity, low contact area, and high unit loading. The form of lubrication normally found in nonconformal contacts is elastohydrodynamic lubrication. Depending on the effects of the elastic deformation and the variation of the viscosity with pressure, four regimes of lubrication can be defined, isoviscous-rigid, isoviscous-elastic, piezoviscous-rigid, and piezoviscous-elastic. In this literature, several aspects of lubrication of nonconformal contacts are presented.

The first objective of this work is the development of representations of minimum film thickness results for piezoviscous-rigid regime of lubrication which carries the same measure of confidence as those already available for the other three regimes of lubrication. This does not only enable the minimum film thickness to be calculated with greater accuracy in the piezoviscous-rigid regime. It also completes the map of lubrication regimes of nonconformal contacts. The compressible Newtonian fluid with Roelands viscosity is considered. The extended set of solutions includes geometry effects where radius ratio less than 1 and the influence of lubricant entrainment direction. The results provide a basis for the analysis and design of a wide range of machine elements operating in the piezoviscous-rigid regime of lubrication.

The second major feature of this study is to propose a new numerical method of calculating elastic deformation in contact stresses. The improved elastic deformation calculation uses a biquadratic polynomial to approximate the pressure distribution on the whole domain analyzed, and the deformation of every node is expressed as a linear combination of the nodal pressures whose coefficients can be combined into an influence coefficient matrix. This approach has the advantages of higher numerical accuracy, less computing time and smaller storage size requirement for the influence matrix.

The third purpose of this study is to extend the ideal elastohydrodynamic lubrication to the real bearing systems in order to get a better understanding of the failure mechanism in machine elements. The improved elastic deformation calculation is successfully incorporated into the EHL numerical scheme. Using this revised numerical technique, surface roughness effects on the elastohydrodynamic lubrication of point contact is studied as an attempt to extend the ideal elastohydrodynamic lubrication to the real bearing systems. The flow factor model is adopted to study surface roughness effect on the elastohydrodynamic lubrication of point contact. In this formulation, the actual flow between rough surfaces is equated to an averaged flow between nominally smooth surfaces, while parameters describing the roughness are included in the Reynolds equation through flow factors. The system selected is a compressible isothermal Newtonian fluid with

Roelands viscosity. Conditions typical of an EHL contact in the piezoviscous-elastic regime entrained in pure rolling on rough bearing are investigated. Results are presented demonstrating the effect of surface roughness on the film shape, pressure distribution, and rolling friction in a ball bearing.

Furthermore, the fourth phase of this work consists of experiments designed and conducted to study the transient EHL effects in instrument ball bearings. A parched subregime of elastohydrodynamic lubrication, lying between starved and mixed, is thereby proposed to describe instrument ball bearing behavior. The parched regime is characterized by zero lubricant flow outside the Hertz area, slow crossflow inside the Hertz area and long term transients in film thickness. A parched bearing has the least driving torque demand and best spin axis definition possible from any lubrication regime.

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TABLE 3.1 - EFFECT OF DIMENSIONLESS LOAD PARAMETER ON MINIMUM FILM THICKNESS

Dimensionless load parameter	Minimum film thickness		Percentage Difference
	Obtained from computational result	Obtained from least square fit	
$W \cdot 10^7$	$H_0 \cdot 10^6$	$\tilde{H}_0 \cdot 10^6$	0
0.90971	4.1	4.13	0.77
0.89091	4.2	4.21	0.20
0.85159	4.4	4.38	-0.48
0.80860	4.6	4.78	-0.37
0.77376	4.8	4.76	-0.75
0.74467	5.0	4.93	-1.45
0.68766	5.2	5.29	1.64
0.65859	5.5	5.49	-0.18
0.64956	5.6	5.56	-0.77
0.63293	5.8	5.69	-1.98

TABLE 3.2 - EFFECT OF DIMENSIONLESS SPEED PARAMETER ON MINIMUM FILM THICKNESS

Dimensionless speed parameter	Dimensionless load parameter	Minimum Film Thickness		Percentage difference
		Obtained from computational result	Obtained from least square fit	
$U \cdot 10^{11}$	$W \cdot 10^6$	$H_0 \cdot 10^6$	$\tilde{H}_0 \cdot 10^6$	0
0.21883	0.11750	4.8	4.68	-2.56
0.20120	0.11009	4.8	4.83	0.72
0.18516	0.88950	4.8	4.84	0.76
0.16833	0.77377	4.8	4.85	0.96
0.15150	0.67629	4.8	4.77	-0.53
0.13466	0.58454	4.8	4.68	-2.58

TABLE 3.4 - EFFECT OF DIMENSIONLESS MATERIALS PARAMETER ON MINIMUM FILM THICKNESS

Solid material	Lubricant	Dimensionless parameter			Minimum film thickness		Percentage difference
		Materials	Speed	Load	Obtained from computational result	Obtained from least square fit	
		G	$U \cdot 10^{11}$	$W \cdot 10^7$	$H_0 \cdot 10^6$	$\tilde{H}_0 \cdot 10^6$	D
Steel	Paraffinic	4522	0.16832	0.77376	4.8	4.681	-2.48
Bronze	Paraffinic	2310	0.32957	1.51499	4.8	4.682	-2.45
Silicon nitride	Paraffinic	6785	0.11218	0.51571	4.8	4.682	-2.45
Steel	Naphthenic	7031	0.48123	1.18602	1.4	1.441	2.92
Bronze	Naphthenic	3591	0.94227	2.32216	1.4	1.441	2.92
Silicon nitride	Naphthenic	10549	0.32073	0.79047	1.4	1.441	2.92

TABLE 3.3 - EFFECT OF ELLIPTICITY PARAMETER ON MINIMUM FILM THICKNESS

Radius ratio	Dimensionless load parameter	Minimum film thickness		Percentage difference
		Obtained from computational result	Obtained from least square fit	
α	$W \cdot 10^6$	$H_0 \cdot 10^6$	$\tilde{H}_0 \cdot 10^6$	D
64	1.61064	4.8	4.805	0.10
61	1.59341	4.8	4.804	0.09
59	1.56632	4.8	4.828	0.58
58	1.55278	4.8	4.842	0.89
54	1.49756	4.8	4.900	2.09
49	1.42381	4.8	4.969	3.52
42	1.13262	4.8	5.044	5.09
34	1.17520	4.8	5.065	5.52
30	1.09985	4.8	5.039	4.99
27	1.04072	4.8	4.994	4.04
20	0.88157	4.8	4.669	-2.83
18	0.83509	4.8	4.691	-2.27
16	0.77377	4.8	4.615	-3.84
12	0.60153	4.8	4.636	-3.42
10	0.53541	4.8	4.764	-7.57
8	0.45598	4.8	4.236	-11.66
4	0.28576	4.8	3.445	-28.22
1	0.12031	3.0	1.953	-34.91
0.8	0.09028	3.0	2.019	-32.70
0.6	0.09548	2.0	1.447	-27.65
0.4	0.05424	2.0	1.593	-20.36
0.2	0.02073	1.5	1.874	24.23

TABLE 3.5 - EFFECT OF LUBRICANT ENTRAINMENT DIRECTION ON MINIMUM FILM THICKNESS

Lubricant entrainment direction	Dimensionless load parameter	Minimum film thickness		Percentage difference
		Obtained from computational result	Obtained from least square fit	
θ	$W \cdot 10^6$	$H_0 \cdot 10^6$	$\bar{H}_0 \cdot 10^6$	D
0	0.14965	5.6	5.60	-0.00
5	0.14841	5.6	5.56	-0.69
10	0.14532	5.6	5.54	-1.03
15	0.14043	5.6	5.52	-1.49
20	0.13427	5.6	5.50	-1.80
25	0.12708	5.6	5.50	-1.80
30	0.11941	5.6	5.54	-1.14
35	0.11902	5.6	5.60	-0.00
40	0.10200	5.6	5.49	-1.94
45	0.09278	5.6	5.33	-4.88

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TABLE 3 - DATA SHOWING EFFECT OF LEAD, SPEED, RADIUS RATIO MATERIAL PARAMETER AND LUBRICANT ENTRAINMENT DIRECTION ON
MINIMUM FILM THICKNESS

Case	Dimensionless load parameter	Dimensionless speed parameter	Radius ratio	Dimensionless material parameter	Lubricant entrainment direction	Minimum film thickness		Percentage difference	Results
						Obtained from computational results	Obtained from Leall square fit		
	$W \cdot 10^7$	$U \cdot 10^{11}$	a	ξ	ϕ	$H_0 \cdot 10^6$	$H_0 \cdot 10^6$	D	
1	0.00471	0.16033	16	4522	0	4.1	4.13	0.32	Load
2	0.00406					4.2	4.21	0.24	
3	0.05159					4.4	4.38	-0.47	
4	0.00406					4.6	4.58	-0.36	
5	0.77376					4.6	4.76	-0.74	
6	0.74467					5.0	4.93	-1.44	
7	0.68766					5.2	5.11	-1.65	
8	0.65059					5.5	5.45	-0.17	
9	0.64956					5.6	5.56	-0.76	
10	0.63753					5.8	5.68	-1.57	
11	1.17505	0.21063				4.8	4.60	-4.20	Speed plus case 5
12	1.0085	0.20120					4.75	-0.97	
13	0.88356	0.18516					4.35	-0.94	
14	0.67625	0.15750					4.69	-2.26	
15	0.58454	0.13466					4.60	-4.22	
16	1.51459	0.32957		2310			4.64	-0.33	Material plus case 5
17	0.9357	0.17218		4785			4.64	-0.33	
18	0.11860	0.48720		7031		1.4	1.38	-2.14	
19	2.32216	0.8422		3591			1.38	-2.14	
20	0.78047	0.32073		10549			1.38	-2.14	
21	1.61064	0.16833	54	4522		4.6	4.96	3.32	Radius ratio plus case 5
22	1.58347		61				4.95	3.13	
23	1.56632		58				4.98	3.82	
24	1.55276		58				4.95	4.14	
25	1.49756		54				5.06	5.37	
26	1.42387		49				5.13	6.85	
27	1.3262		42				5.21	8.47	
28	1.17520		34				5.22	8.92	
29	1.09865		30				5.20	8.37	
30	1.0402		27				5.15	7.36	
31	0.88157		20				4.94	2.92	Lubricant entrainment direction
32	0.83509		18				4.84	0.88	
33	0.80753		12				4.77	-0.72	
34	0.5354		10				4.44	-7.64	
35	0.45598		8				4.38	-8.87	
36	0.28576		4				3.56	-25.9	
37	0.1203					3.0	2.02	-32.82	
38	0.09528		0.8			3.0	2.06	-30.53	
39	0.05446		0.6			2.0	1.50	-25.23	
40	0.05424		0.4			2.0	1.66	-17.20	
41	0.02073		0.2			1.5	1.94	28.65	Lubricant entrainment direction
42	1.49845	0.75745	20		0	5.6	5.51	1.60	
43	1.48407				5		5.55	0.86	
44	1.45276				10		5.53	1.25	
45	1.4042				15		5.50	1.79	
46	1.34264				20		5.49	1.96	
47	1.27083				25		5.48	2.14	
48	1.19406				30		5.52	1.42	
49	1.10916				35		5.56	0.30	
50	1.02001				40		5.57	-1.65	
51	0.92775				45		5.54	-4.64	
52	0.10874	0.25745	16		0	5.8	5.89	1.55	
53	0.1400	0.43677	20		0	15.5	14.5	-6.45	

TABLE 3.7 - COMPARISON BETWEEN
THE PRESENT RESULT AND THE
EQUATION PROPOSED
BY HOUPERT

Case	$\frac{H_o \text{ (Houpert)}}{H_o \text{ (Jeng)}}$
1	1.52
2	1.50
3	1.42
4	1.38
10	1.20
11	1.54
12	1.46
13	1.40
16	1.34
17	1.34
18	1.28
19	1.28
20	1.28
21	1.37
24	1.36
30	1.35
34	1.36
37	0.969
38	0.968
39	0.927

TABLE 4.1 - COMPARISON OF THE ELASTIC DEFORMATIONS
OBTAINED WITH TWO METHODS (UNIT CM)

[$E = 21.97 \text{ MN/cm}$; $R_x = R_y = 0.5558 \text{ cm}$; $F = 8.964 \text{ N}$]

NX	NY	$\frac{d_{\max}}{d_{\text{Hep}}} - 1$		Δh_{\max}	
		H D	New Approach	H D	New Approach
15	15	1.183×10^{-2}	1.225×10^{-3}	1.652×10^{-6}	8.173×10^{-7}
19	19	6.329×10^{-3}	2.161×10^{-4}	1.264×10^{-6}	6.042×10^{-7}
31	31	2.448×10^{-3}	1.320×10^{-5}	6.888×10^{-7}	3.042×10^{-7}

H D: Hamrock and Dowson (1974)

TABLE 5.1 - MATERIAL AND LUBRICANT PROPERTIES FOR THE
ILLUSTRATIVE EXAMPLES

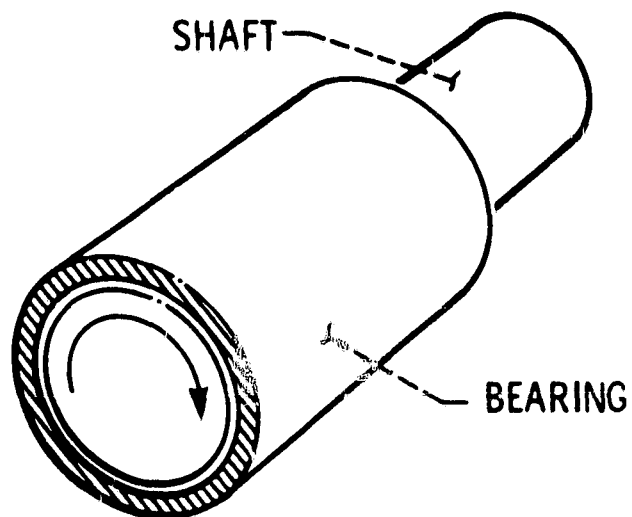
ELASTIC MODULUS OF STEEL BALLS, E	2.00×10^{11}	N/m^2
POISSON'S RATIO FOR STEEL, ν	0.3	
INLET VISCOSITY OF PARAFFINIC LUBRICANT, η_0	4.11×10^{-2}	Ns/m^2
CONSTANTS IN ROELANDS MODEL, η_∞	6.31×10^{-5}	Ns/m^2
γ_c	1.96×10^8	N/m^2
ROELANDS VISCOSITY-PRESSURE INDEX, Z	0.67	
EFFECTIVE BALL RADIUS, R_x	1.11125×10^{-2}	m
SPECIFIC LOADING OF BALLS, F	8.5	N
ROLLING SPEED, u_x	1.0×10^{-1}	m/s
DIMENSIONLESS EHL PARAMETERS, U	1.683×10^{-12}	
W	3.134×10^{-7}	
G	4522.2	
H_{mo}	6.4237×10^{-6}	
k	6	
θ	0	

TABLE 6.1 - 108 BEARING GEOMETRY

Bore	0.04 m (1.57 in.)
Ball diameter	9.53×10^{-3} m (0.375 in.)
Pitch diameter	5.273×10^{-2} m (2.076 in.)
Contact angle	15.67 degrees
Inner cross race curvature	0.54
Outer cross race curvature	0.58
Ball complement	17

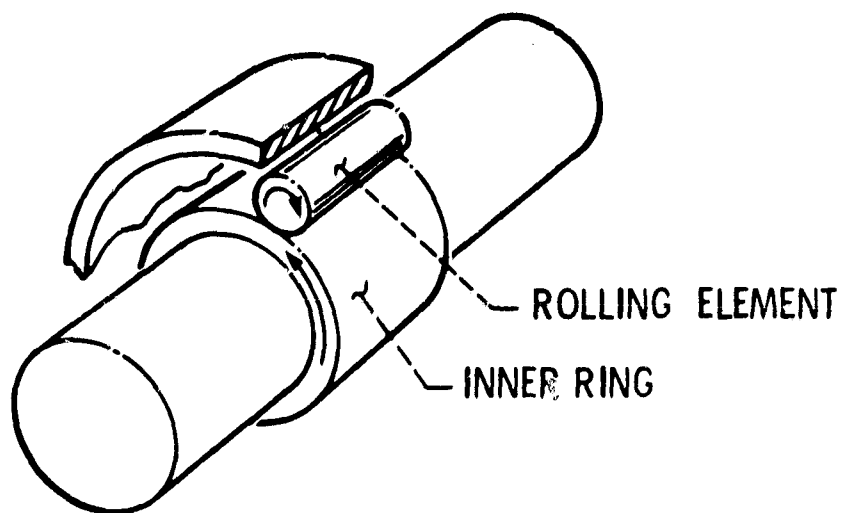
TABLE 6.2 - BEARING OPERATING PARAMETERS

Mean Hertz stress	1×10^9 N/m (150000 psi)
Inner major Hertz width	1.38×10^{-3} m (0.054 in.)
Inner ellipticity ratio	6.16
Outer major Hertz width	1.01×10^{-3} m (0.04 in.)
Outer ellipticity ratio	3.26
Nominal ball spin rate	167 Hz (10000 rpm)
Nominal inner race rate	36 Hz (2160 rpm)
Nominal outer race rate	26 Hz (1560 rpm)
Total ball-race pivoting rate	6.13 Hz (363 rpm)
Entrainment velocity	5 m/s (197 in/s)



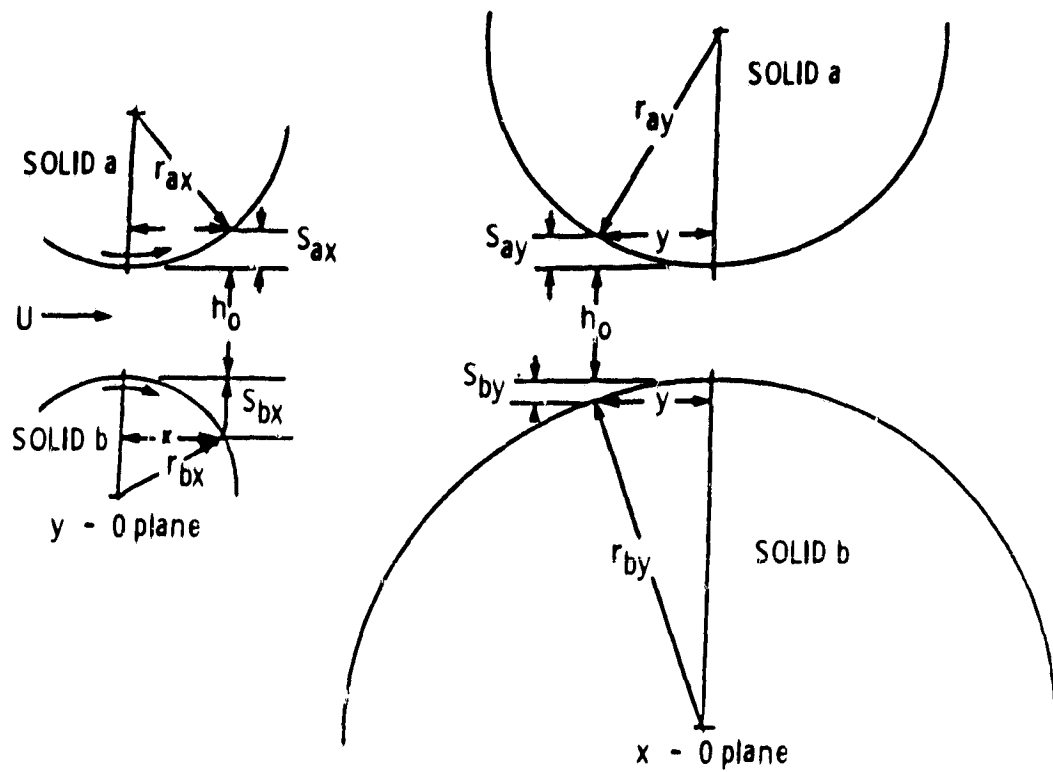
SLIDING SURFACE BEARING. HIGH
CONFORMITY. LARGE EFFECTIVE CON-
TACT AREA AND LOW UNIT LOADING

Figure 1.1. - Conformal surfaces.

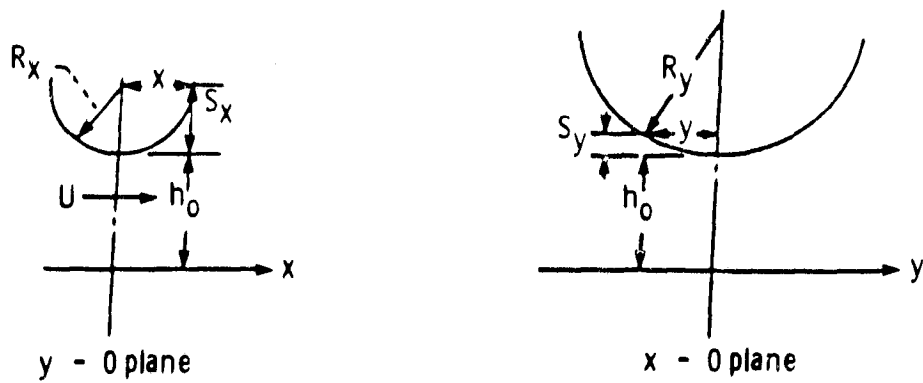


ROLLING ELEMENT BEARING. POOR CONFORMITY,
SMALL CONTACT AREA, HIGH UNIT LOADING

Figure 1. 2. - Nonconformal surfaces.



(a) Two rigid solids separated by a lubricant film.



(b) Equivalent system of a rigid solid near a plane separated by a lubricant film.

Figure 2.1 Contact geometry.

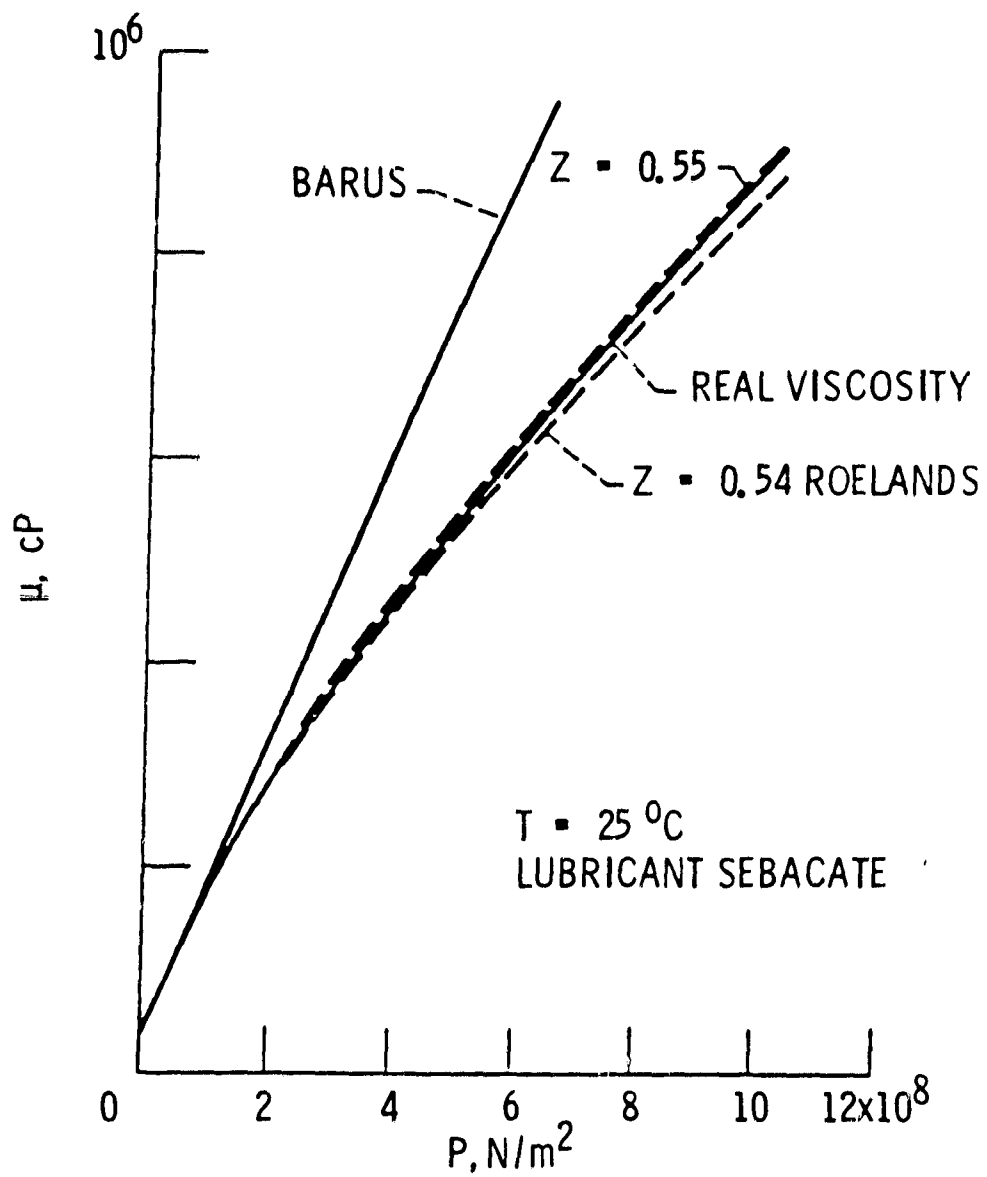


Figure 3.1 Viscosity as a function of pressure.

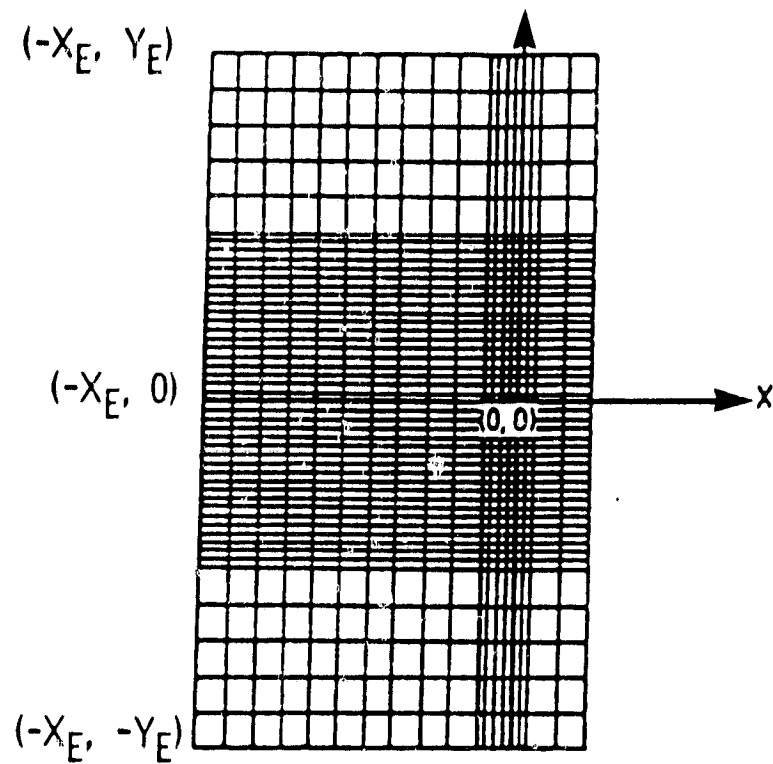


Figure 3.2 Variable nodal structure used for numerical calculations.

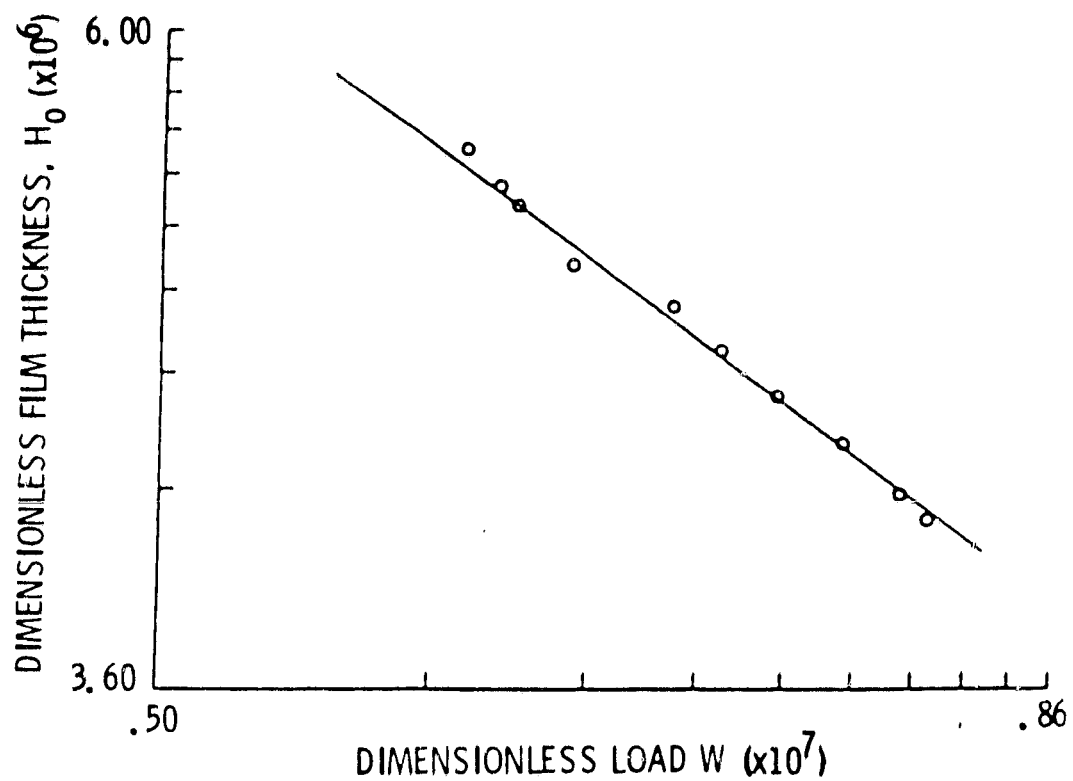


Figure 3.3 The variation of dimensionless minimum film thickness with dimensionless load.

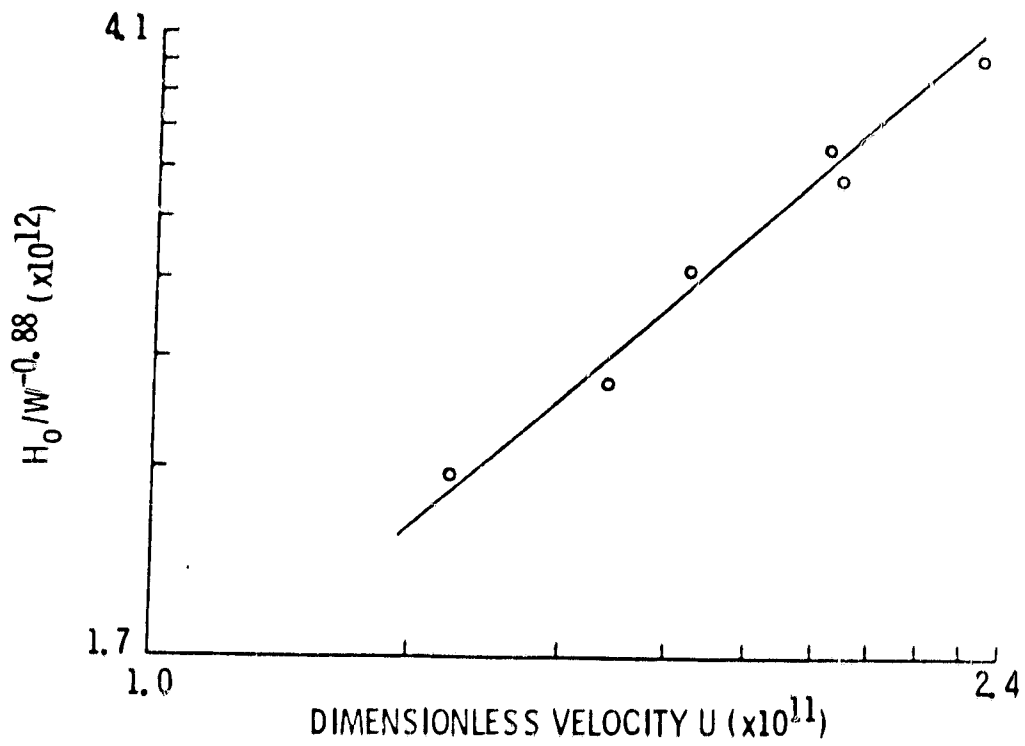


Figure 3.4 The variation of dimensionless minimum film thickness with dimensionless velocity.

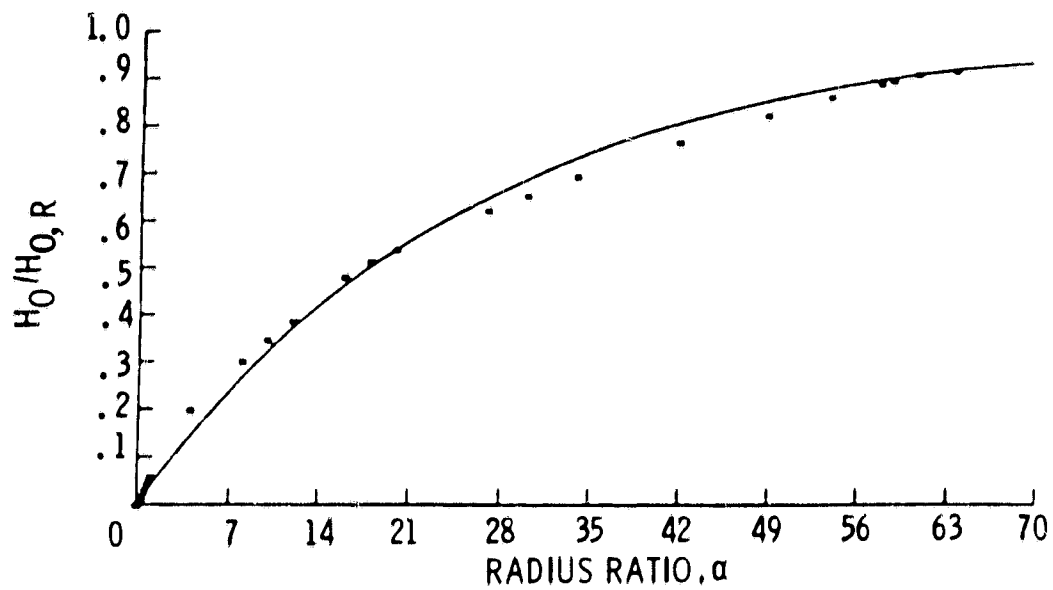


Figure 3.5 The influence of radius ratio on dimensionless minimum film thickness.

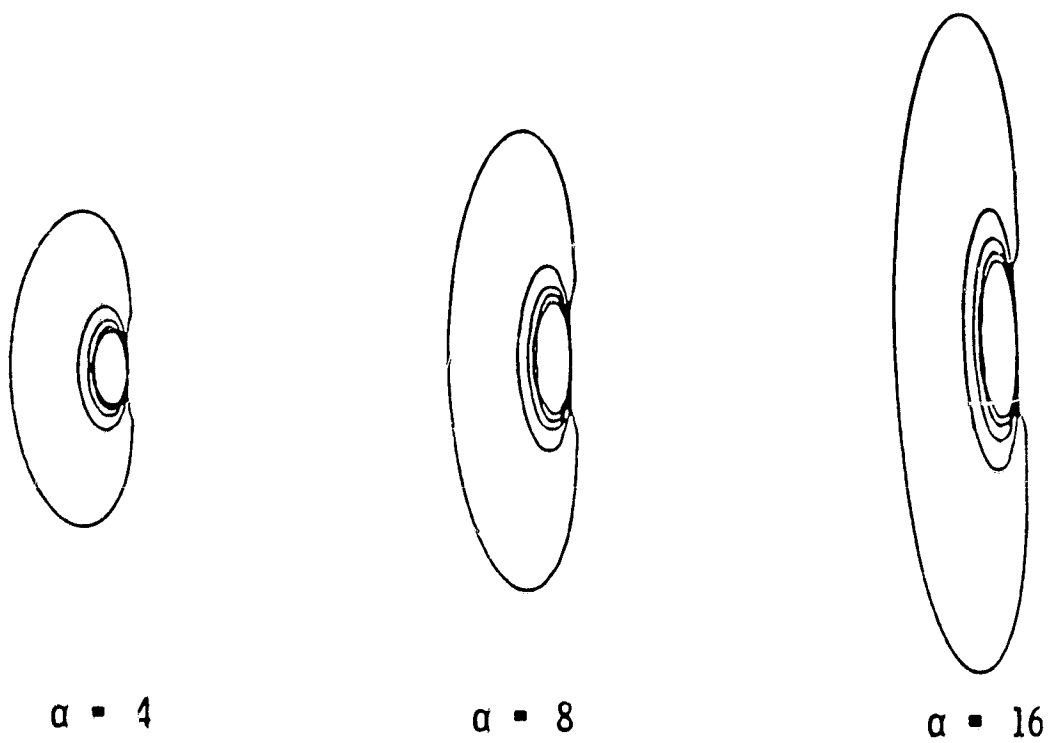


Figure 3.6 Pressure contour for three radius ratios.

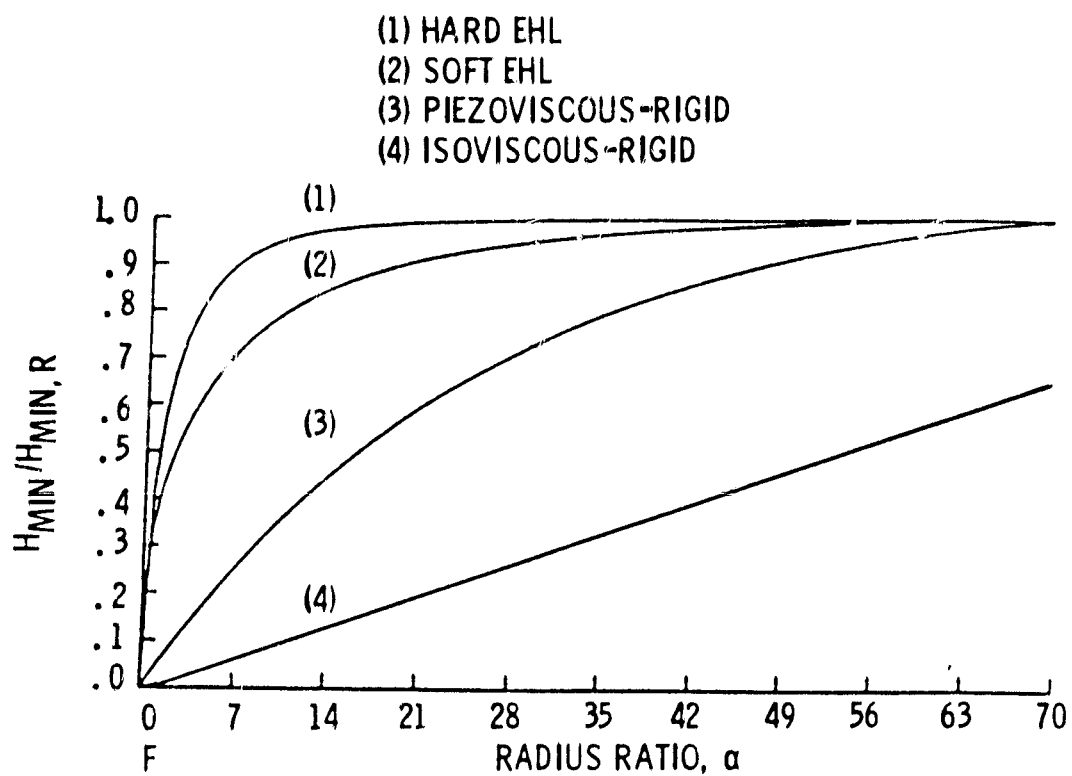


Figure 3.7 Comparison of the geometry effects in four different regimes of lubrication.

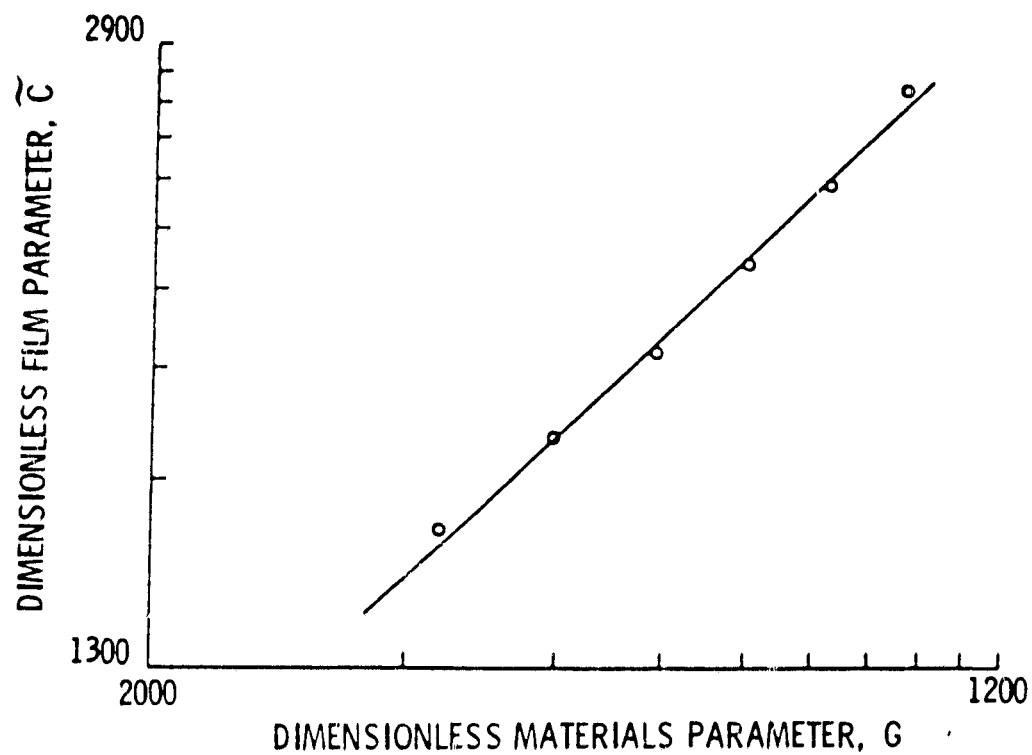


Figure 3.8 The variation of dimensionless minimum film thickness with dimensionless materials parameter.

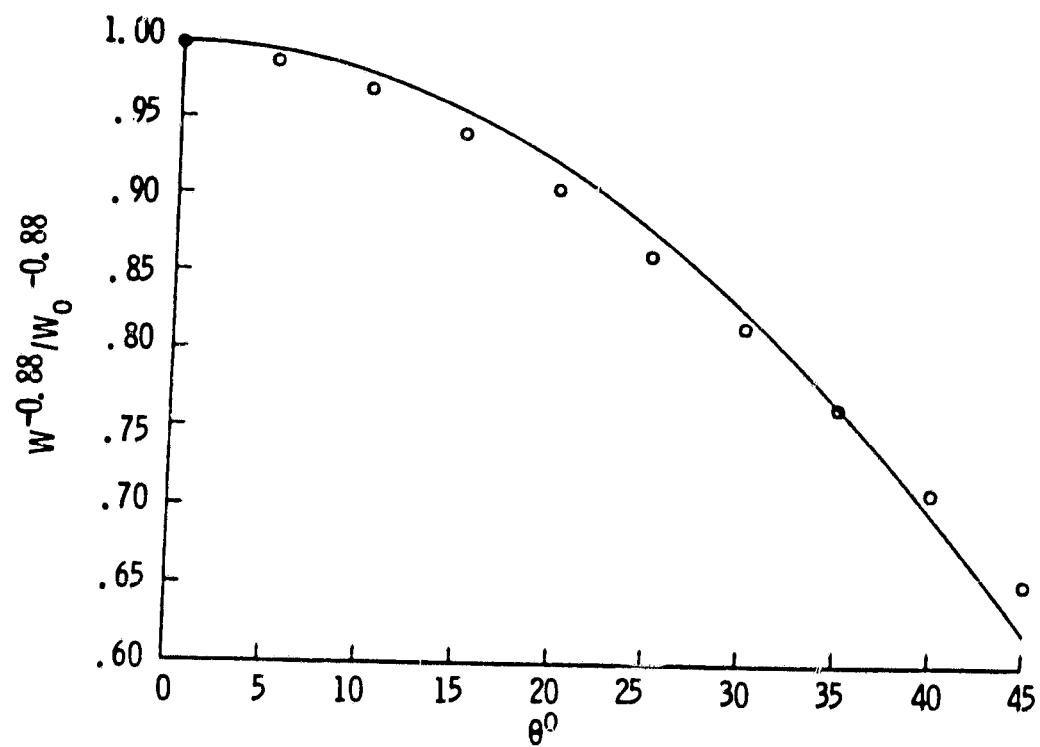


Figure 3.9 The variation of minimum film thickness with the lubricant entrainment direction.

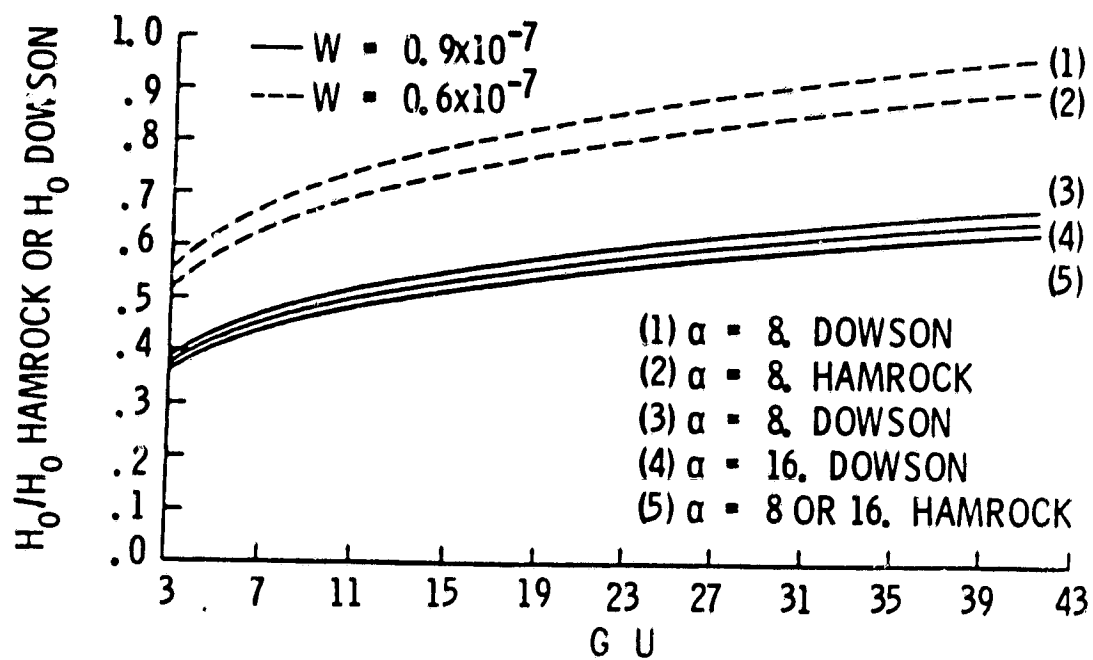
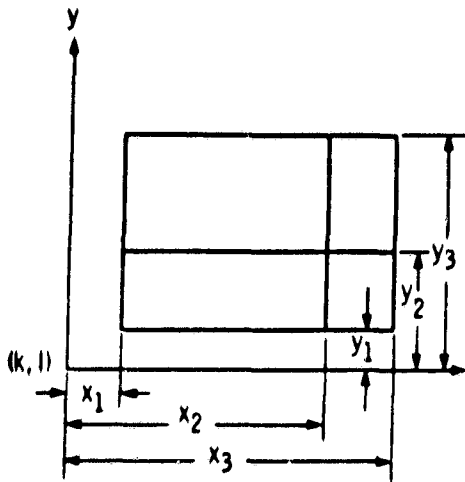
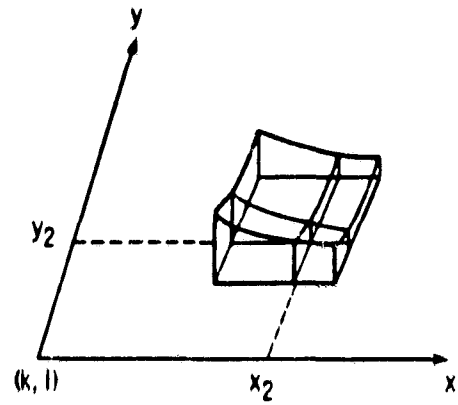


Figure 3.10 Comparison between different minimum film thickness formulae.



(a) Rectangular elements with nine nodes.



(b) Representation of pressure distribution by paraboloidal surfaces.

Figure 4.1 Coordinate system for the computing element.

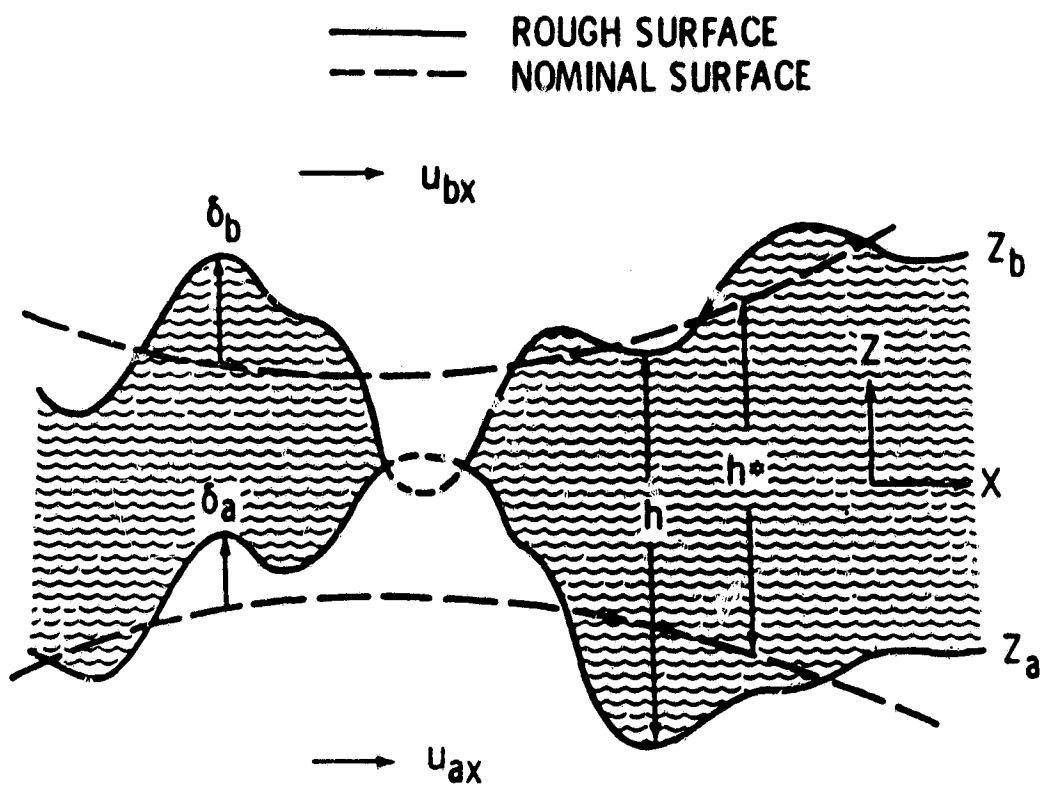
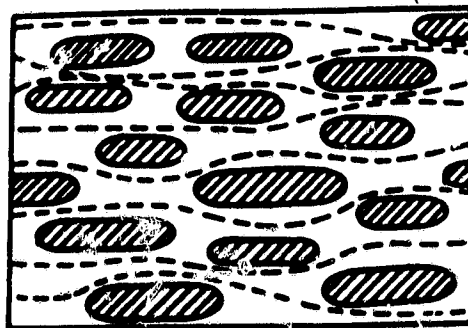
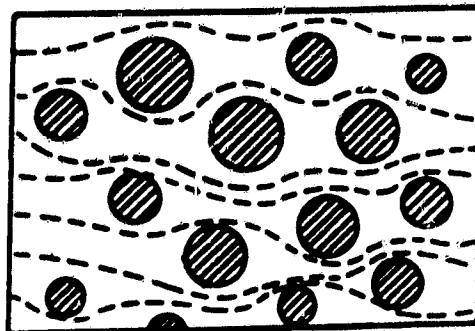


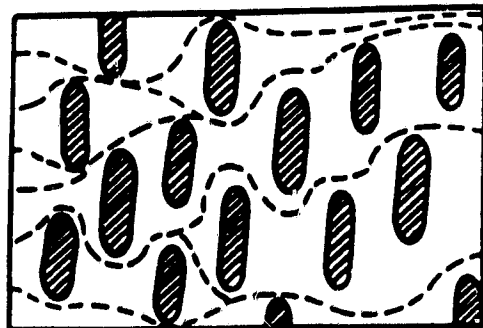
Figure 5.1 Lubrication of moving rough surfaces.



$\gamma > 1$



$\gamma = 1$



$\gamma < 1$

Figure 5.2 Typical contact areas for longitudinally oriented ($\gamma > 1$), isotropic ($\gamma = 1$) and transversely oriented ($\gamma < 1$) surfaces.

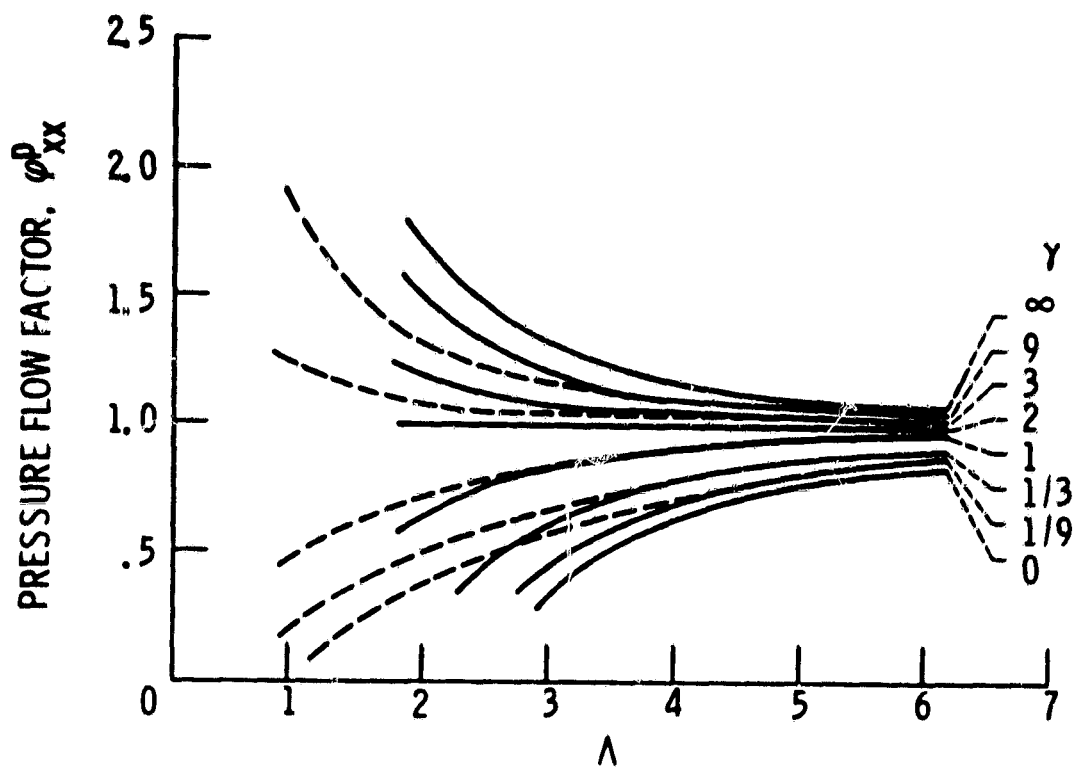


Figure 5.3 Comparison of pressure flow factor calculated from equation (5.7) (solid line) with those of Patir and Cheng (1978) (dashed line).

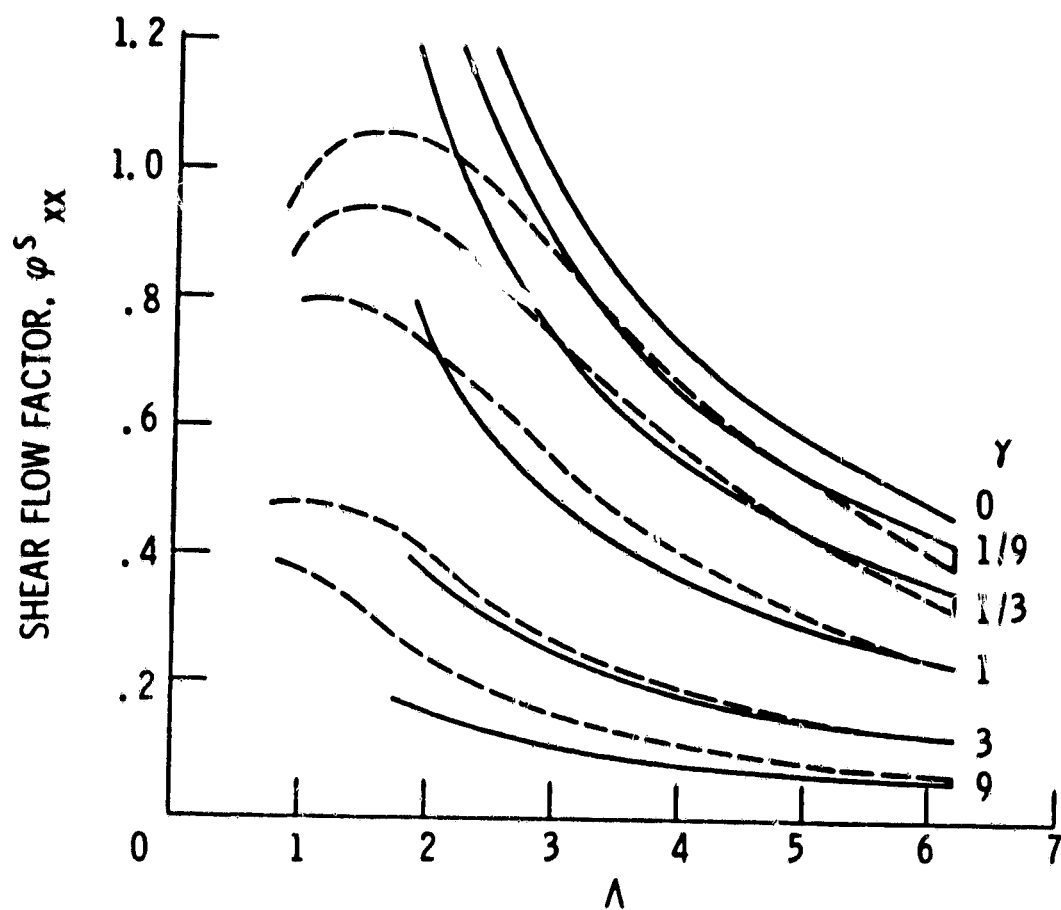


Figure 5.4 Comparison of shear flow factor calculated from equation 5.7 (solid line) with those of Patir and Cheng (1979) (dashed line).

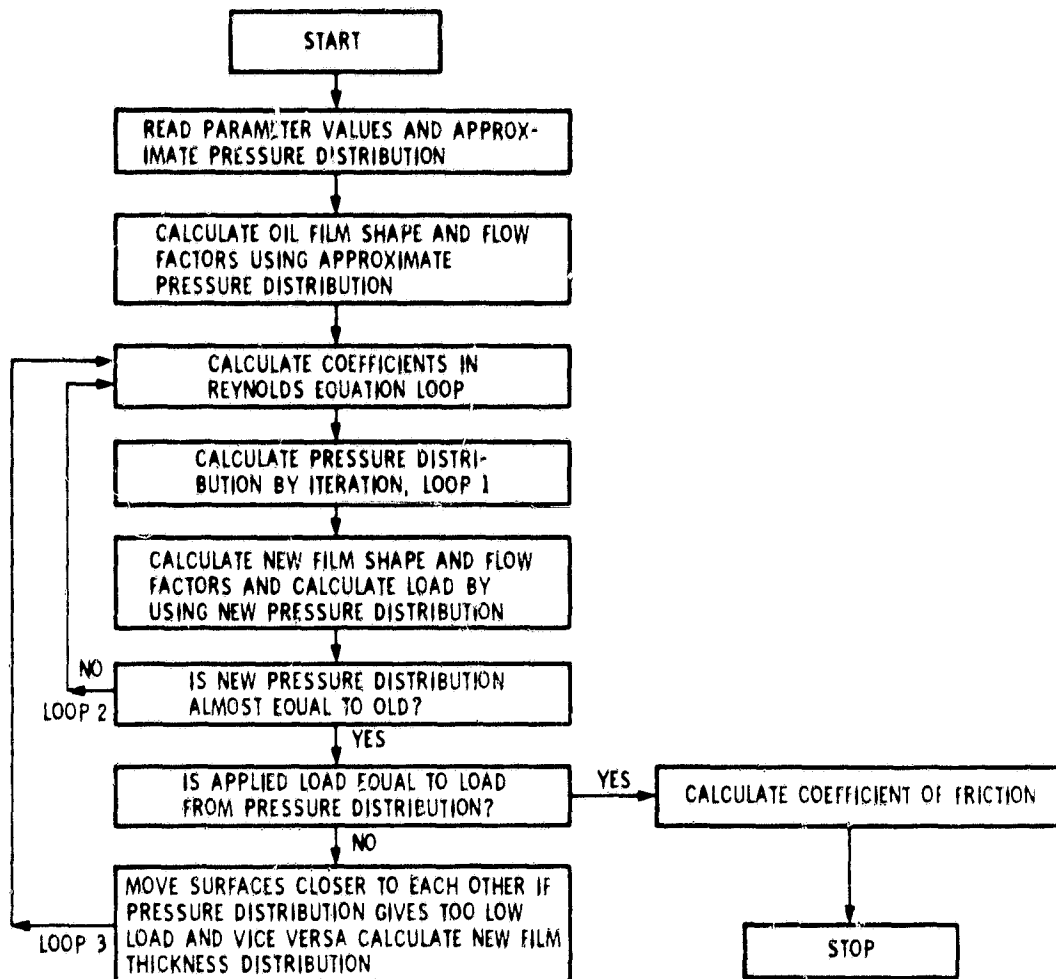


Figure 5.5 - Flow chart of computer code.

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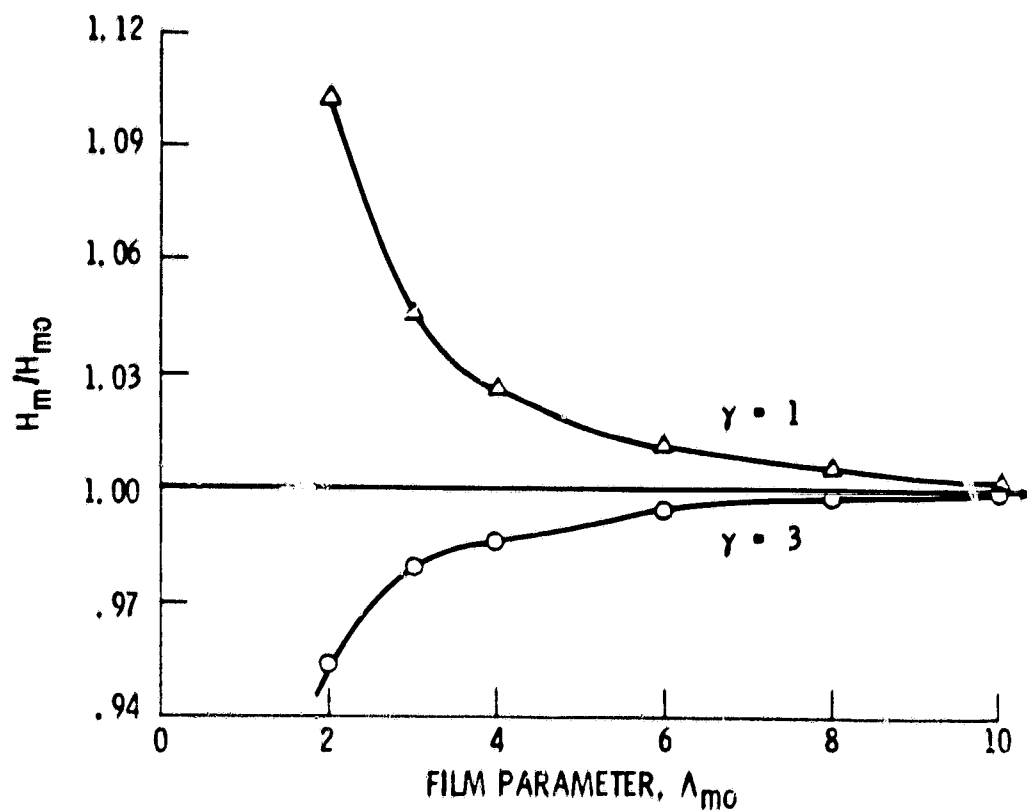


Figure 5.6 Variation of minimum film thickness at constant load with film parameter for different asperity anisotropy.

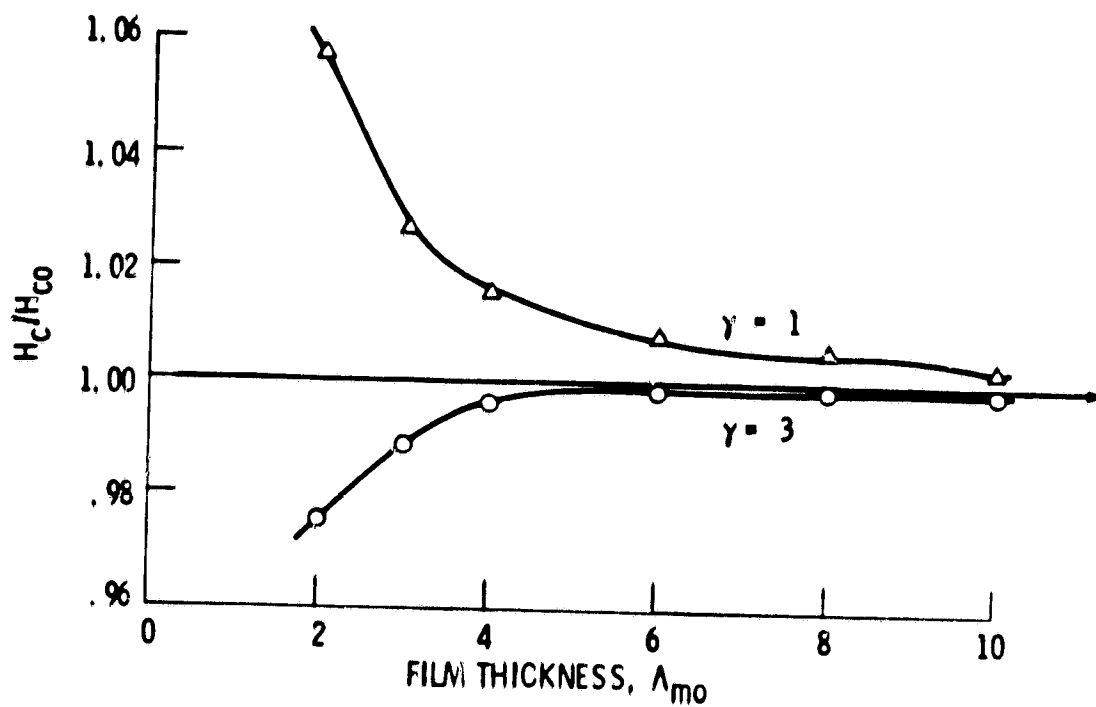


Figure 5.7 Variation of central film thickness at constant load with film parameter for different asperity anisotropy.

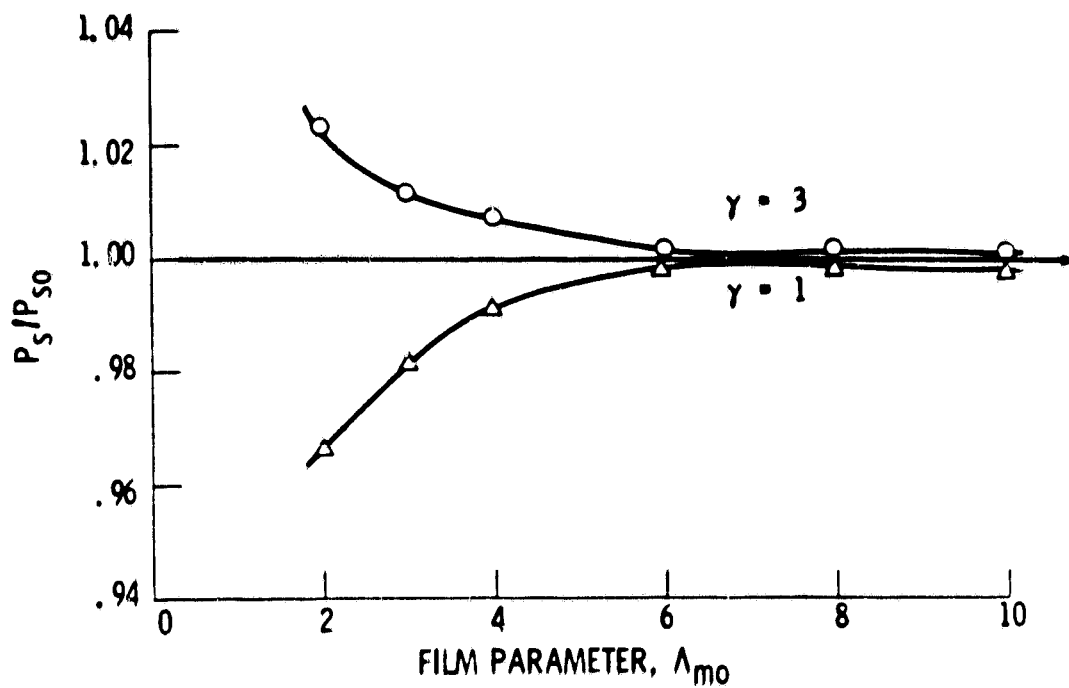


Figure 5.8 Variation of pressure spike at constant load with film parameter for different asperity anisotropy.

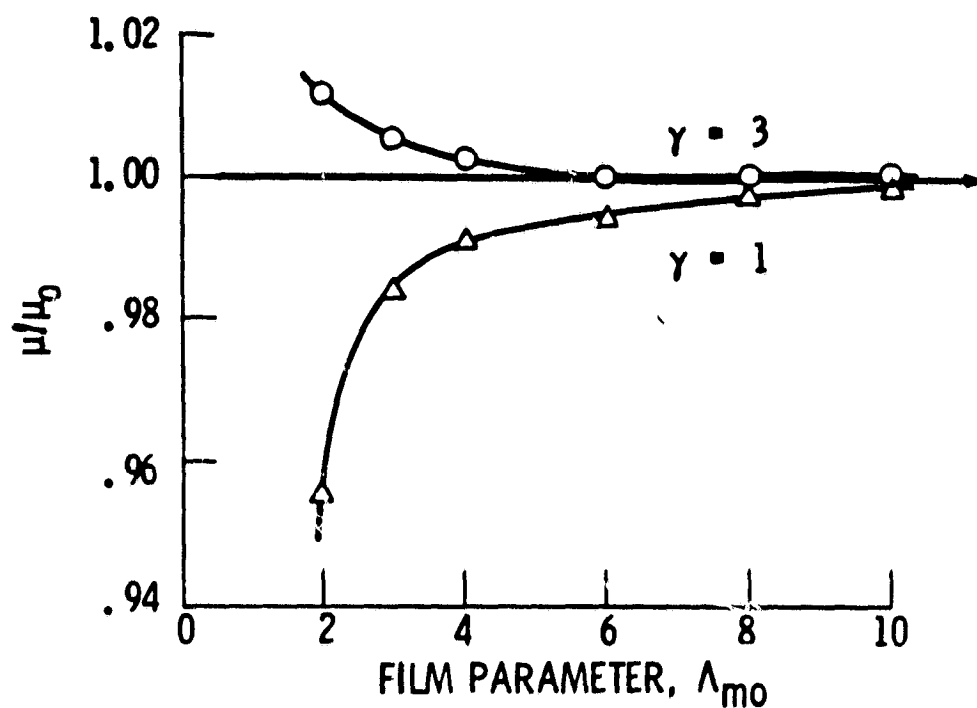


Figure 5.9 Variation of rolling friction coefficient at constant load with film parameter for different asperity anisotropy.

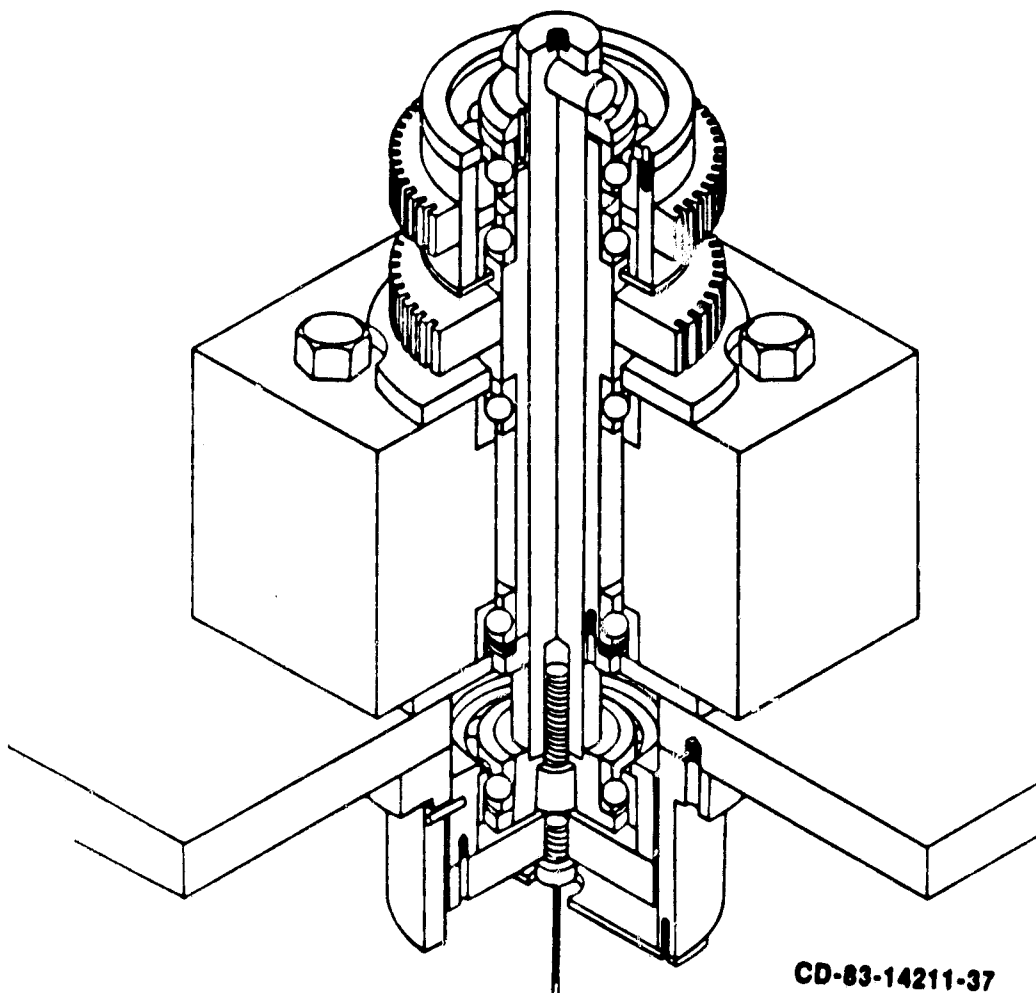


Figure 6.1 Apparatus.

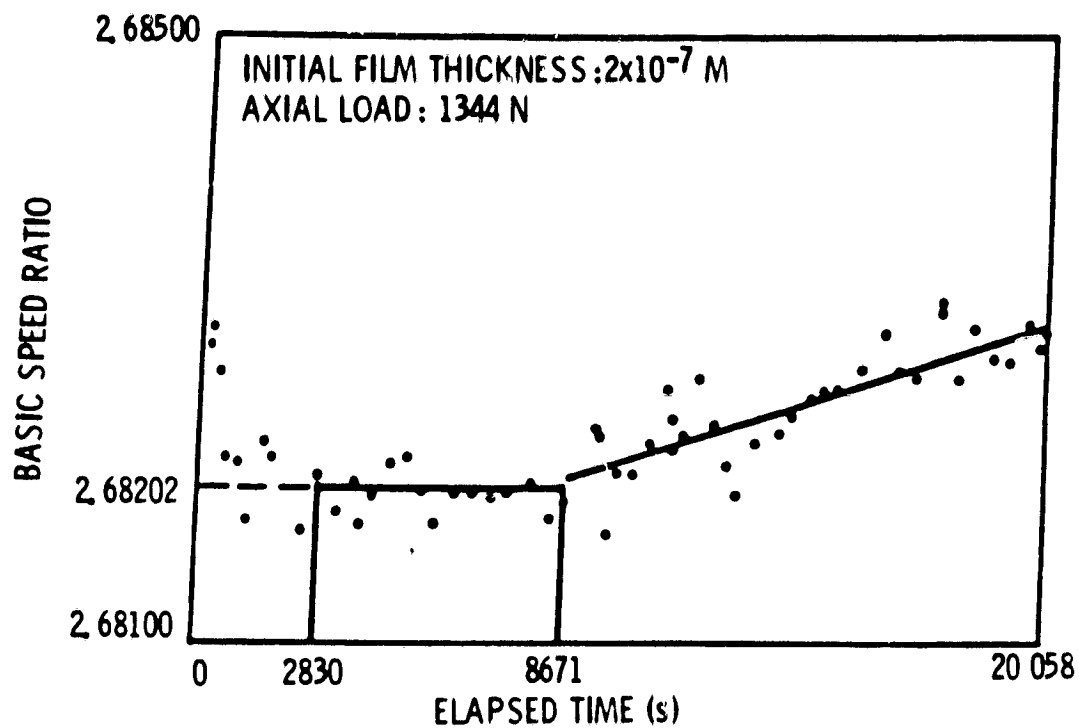


Figure 6.2 Basic speed ratio versus time (thick film).

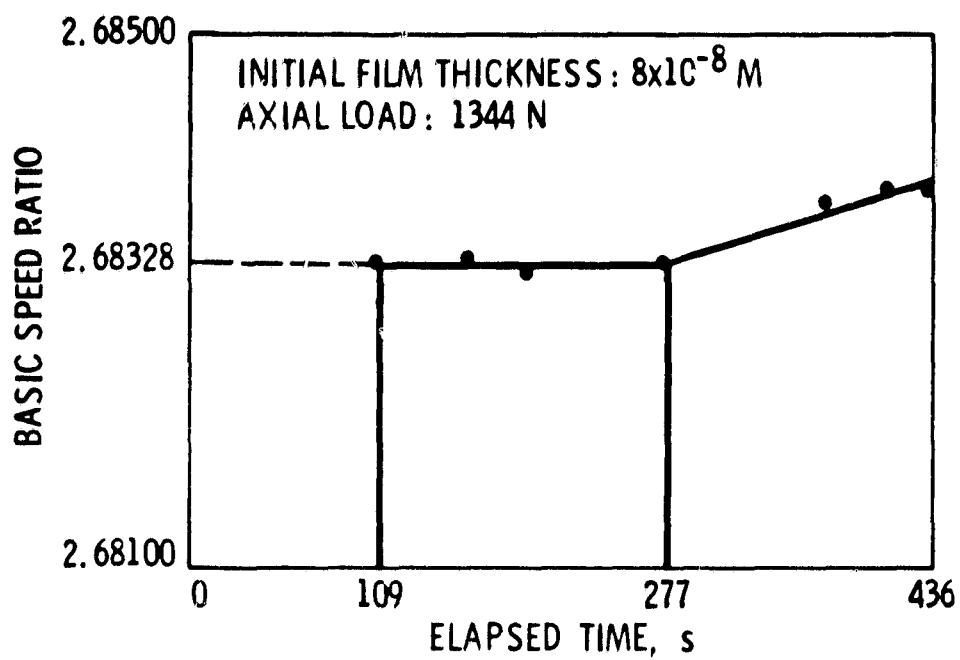


Figure 6.3 - Basic speed ratio versus time (thin film).

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APPENDIX 1

THE VARIABLE-MESH NODAL STRUCTURE

A variable-mesh nodal structure was used to provide close spacing in and around the pressure spike. This help to minimize the errors that can occur because of large gradients in the high-pressure region. The locations of the pressure spikes from Kapitza (1955) are determined by setting $(\partial P/\partial X) = (\partial P/\partial Y) = 0$ and solving for X and Y as follows:

$$X_{pk} = -\sqrt{\frac{2H_0}{3}}$$

$$Y_{pk} = 0$$

The fine spacing are determined so that there are 2 or 3 nodes between H_0 and X_{pk} . An interative program is developed to set up the variable-mesh nodal structure. The nodal structure used was varied depending on anticipated pressure distribution. A nodal structure is used when a good convergence is observed by changing the nodal structure then it is used for similar pressure distribution.

Appendix 2

COMPUTER PROGRAM LIST 1

PROGRAM NAME	UTILITY
NODAL	Set up the variable-mesh structure
MAIN	Mainprogram used to input the variable-mesh strucutre.
PVRH	subprogram used to analyze the lubrication of piezoviscous-rigid regime.

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300020 C      NODAL
300100      IMPLICIT REAL*8(A-H,O-Z)
300200      NAMELIST /OUT/NX,NY,MCGF,NZON2,JZON,NIM,H0,ZC1,ZC2,ZD1,ZEI,XNODE,XI,YI,NXY,NXPK,NPK
300300      DATA HIN/0.1200/
300400      DATA H0-ALPHA/2.00-5.20.000/
300500      DATA NODE,MCGF/1,12/
300600      DATA ZC1,ZD1/100.000,100.000/
300700      SPACE=DSORT(2./3.*H0)*0.5
300800      XP=1.-SPACE*2.0
300820      XPK=1.11125*XPK
301000      ZE1=NODE/SPACE
301100      WRITE(6,100)FZONE1,FZONE,FCZONE,XF
301200      FORMAT(1X,'INPUT NZON1,NZON2,IFNODE',1X,4D16.8,1X,'(FZONE1,FZONE,FCZONE,XF,INDE)')
301300      READ(5,150)NZON1,NZON2,IFNODE
301400      FORMAT(3I3)
301500      FZONE=IFNODE/ZD1
301600      FZONE1=NZON1/ZE1
301700      FZONE=NZON2/ZE1
301800      XF=1.+FZONE-FZONE1+FCZONE
301900      IF(FZONE1.LE.0.15)GO TO 5
302000      IF(FZONE.LE.0.196)GO TO 5
302100      IF(XF.LE.1.1225)GO TO 5
302200      WRITE(6,110)FZONE1,FZONE,FCZONE,XF,INDE
302300      FORMAT(1X,'INPUT INDE',4D16.8,1X,11)
302400      READ(5,120)INDE
302500      FORMAT(11)
302600      IF(INDE.EQ.1) GO TO 5
302700      CZONE=(MCGF-1)/100.000
302800      XI=1.-FZONE1-CZONE
302900      X=1.-XI
303000      XNODE=XI+CZONE
303100      NX=MCGF+NZON2+IFNODE
303200      NXPK=MCGF+NZON1-NODE
303300      Y=ALPHANDSORT(1.-(1.-1./ALPHAN*(HIN-H0-1.+DSORT(1.-X*X)))**2)
303400      YI=8.4895358500-Y
303500      WRITE(6,200)ZC2,YZONE
303600      FORMAT(1X,'INPUT NY,JZON,IND',2D16.8,'(ZC2,YZONE)')
303700      READ(5,250)NY,JZON,IND
303800      FORMAT(3I3)
303900      NYF=NY-JZON
304000      YZONE=Y-(JZON-1)/ZC1
304100      ZC2=NYF/YZONE
304200      IF(IND.EQ.1)GO TO 6
304300      NXY=NXY*NY
304400      WRITE(6,OUT)
304500      WRITE(4,OUT)
304600      STOP
      END

```

```

000020 C MAIN
000100 IMPLICIT REAL*8(A-H,O-Z)
000200 DIMENSION H(59400),PR(59400)
000300 REAL*4 XE(450),XG(132),YG(132),YGC(899),HS(59400),PRS(59400)
000400 REAL*4 XMI(450)
000500 COMMON/STARTIV/HIN,H0,XNODE,ZC1,ZC2,ZD1,ZE1,XI,YI
000600 COMMON/STARTII/JZON,NCGF,NZON2,IDE,NXPK
000700 INPUT
000800 NX=132
000900 NY=270
001000 NCGF=13
001100 NZON2=110
001200 JZON=60
001300 IDE=1
001400 HIN=0.1500
001500 H0=4.8D-6
001600 ZC1=100.0D0
001700 ZC2=164.00006139334D0
001800 ZD1=100.0D0
001900 ZE1=559.01701103495D0
002000 XNODE=0.83900311041810
002100 XI=0.71900311041810D0
002200 YI=6.6191109841981D0
002300 NXPK=102
002400 M=1
002500 C
002600 C M=-1: READS IN FORMATTED INPUT AND BYPASSES THE INITIAL
002700 C CALCULATION OF THE PRESSURE DISTRIBUTION
002800 C
002900 C M=0: READS IN HEXADECIMAL INPUT AND BYPASSES THE INITIAL
003000 C CALCULATION OF THE PRESSURE DISTRIBUTION
003100 C
003200 C M=+1: CALCULATES AN INITIAL GUESS
003300 C
003400 1 CONTINUE
003500 NX=NX-NY
003600 NYMI=2*NY-1
003700 CALL RIGID3(CNX,NY,MXY,NYMI,H,PR,PRS,HS,XG,YG,YGC,XE,XMI,M)
003800 STOP
003900 END

```

```

100020 C
00100 SUBROUTINE RIGIDJCNX,NY,NXY,NYMI,H,PR,PRS,HS,XG,YG,XE,XMI,M)
00200 IMPLICIT REAL*8(A-H,O-Z)
00300 REAL*4 XMIN,XLEN,YMIN,XE(NY),XMI(NY)
00400 REAL*4 XG(NX),YCNX),YGC(NYMI),HS(NXY),PRS(NXY)
00500 REAL*4 FM,US,MS,ALPHA,MI,XMAX,XCENTS,YCENTS,YMAX,PRMXS,XPK
00600 REAL*4 XNODFS,XDEL,YDEL,HMINS,PHIN,HMAX
00700 NAMELIST/INPUT/RAX,RAY,RBX,RBY,RX,RY,SMU,SMV,PPEE,PL
00800 NAMELIST/RADIUS/RAX,RAY,RBX,RBY,RX,RY,SMU,SMV,PPEE,ALPHA,NXPX,XI,YI
00900 NAMELIST/OUTPUT/PBAR,PCHECK,FBAR,FCHECK,SUM3,MO,HIN,MOU
01000 DIMENSION DENS(6680),VIS(6680),XMU(6680),ZPR(6680),PHI(6680)
01100 DIMENSION A(6680),B(6680),C(6680),DLZ(6680),XL(6680)
01200 DIMENSION XM(6680)
01300 DIMENSION S(6680),H(NXY),PR(NXY)
01400 COMMON/STARTV/HIN,MO,XNODE,ZC1,ZC2,ZD1,ZEL,XI,YI
01500 COMMON/STARTI/JZON,MCGF1,MZON2,IDE,HXPK
01600 COMMON/CTRINF/FN,US,MS,ALPHA,MI,XMAX,XCENTS,YCENTS,YMAX,PRMXS,XPK
01700 COMMON/CTR/XNODFS,XDEL,YDEL,HMINS,PHIN,HMAX
01800 COMMON/CTRINT/MCGF
01900 REAL*4 FMTX,PRD(305,999)
02000 C
02100 C INPUT
02200 C
02300 C MPARA=0
02400 C NVISC=0
02500 C MJT=0
02600 C
02700 C MPARA=1: THE PARABOLIC APPROXIMATION WILL BE USED
02800 C NVISC=0: THE ISO-VISCOUS SOLUTION IS DETERMINED
02900 C MJT=1: PARTIALLY CONVERGED SOLUTION IS STORED ON TAPE IN CASE OF
03000 C CRASH
03100 C M=1
03200 C ROSH=1.000
03300 C MAUR=0
03400 C JENN=0
03500 C ZZZ=1.000
03600 C PI=3.1415926535900
03700 C PIVAS=4858.700
03800 C VIS0=4.11D-6
03900 C VA=0.000
04000 C VB=0.000
04100 C Z=.67D0
04200 C ELPHA=5.82746D-6
04300 C BETA=1.68348D-3
04400 C

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004500 VISE=.00000000631D0
004600 ORF=1.9D0
004700 ZE2=ZD1
004800 MCGF=MCGF1
004900 XCEN=1.00D0
005000 YCENT=8.48953585D0
005100 C
005200 C M=-1: READS IN FORMATTED INPUT AND BYPASSES THE INITIAL
005300 C CALCULATION OF THE PRESSURE DISTRIBUTION
005400 C
005500 C M=0: READS IN HEXADECIMAL INPUT AND BYPASSES THE INITIAL
005600 C CALCULATION OF THE PRESSURE DISTRIBUTION
005700 C
005800 C M=+1: CALCULATES AN INITIAL GUESS
005900 C
006000 C CONTINUE
006100 C DEL1=.1D0
006200 C MAXS2=1
006300 C DEL3=.1D0
006400 C HMIN=10.0D0
006500 C HMAX=10.0
006600 C PRSMX=.1D-18
006700 C PRMX=.1D-18
006800 C CONTINUE
006900 C HIN=HIN
007000 C HX1=HX-1
007100 C NY1=2*NY
007200 C HI=HIN
007300 C READ(7,INPUT)
007400 C
007500 C CURVATURE SUM AND DIFFERENCE
007600 C
007700 C RX=(1./RAX)+(1./RBX)
007800 C RY=(1./RAY)+(1./RBY)
007900 C RHO=RX+RY
008000 C GAMMA=(RX-RY)/RHO
008100 C ALPHA=RX/RY
008200 C
008300 C XCENTS=XCENT/RX
008400 C YCENTS=YCENT/RX
008500 C XNODFS=XNODF/RX
008600 C YM=YCENT
008700 C XN=XCENT
008800 C
008900 C DIMENSIONLESS PARAMETER GROUPING

```

```

009000 C
009100 V=SQRT(SMU**2+SMV**2)
009200 THETA=ATAN(SMV/SMU)
009300 U=VISORRXV/EP
009400 US=U
009500 HO=HO
009600 PHEE=1./(1.+(2.*RY)/(3.*RX))
009700 Q1=VISE/VISO
009800 Q2=EP/1968.5268
009900 Q3=ELPHAE/EP
010000 Q6=BETAE/EP
010100 CONTINUE
010200 ZC=ZC1
010300 SA=1./ZC
010400 SB=1.6/ZD1
010500 C
010600 C INITIAL GUESS
010700 C
010800 IF (M) 5,6,7
010900 CONTINUE
011000 READ(5,700) (PR(N),N=1,NXY)
011100 READ(5,700) (HC(N),N=1,NXY)
011200 FORMAT(8D10.5)
011300 GO TO 7
011400 CONTINUE
011500 READ(5) (PR(N),N=1,NXY)
011600 CONTINUE
011700 Y=Y1
011800 DO 13 J=1,MY
011900 X=X1
012000 DO 12 I=1,MX
012100 ZD=ZD1
012200 N=I+(J-1)*MX
012300 JTMN=NY1-J
012400 IF(J.GE.JZOM) ZC=ZC2
012500 YGC(J)=Y/RX
012600 YGC(JTMN)=Z.*YCENST-YGC(J)
012700 XG(I)=X/RX
012800 NVMF=NCGF+(J-1)*NX
012900 NVMB=NVMF+NZON2
013000 IF(M.GE.NVMB) AND. M.LT.NVMB) ZD=ZE1
013100 IF(M.GE.NVMB) ZD=ZE2
013200 SQX=((X-XN)/RX)**2
013300 SQY=((Y-YN)/RX)**2
013400 IF (MPARA.EQ. 1) GO TO 75
013500 S(K)=(1./RX)+(1./RY)-DSQRT((1./RX)**2-SQX)-DSQRT((1./RY)**2-SQY)

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113600 GO TO 8
113700 S(N)=(RX*SQX+RY*SQY)/2.
113800 CONTINUE
113900 IF(M.EQ.1) GO TO 9
114000 GO TO 10
114100 CONTINUE
114200 PR(N)=.1D-8
114300 CONTINUE
114400 IF (J.EQ.1 .OR. I.EQ.1 .OR. I.EQ.NX) PR(N)=0.000
114500 IF (PR(N).GT.0.) GO TO 11
114600 PR(N)=0.D0
114700 CONTINUE
114800 IF (J.EQ.NY) YG(I)=PR(N)
114900 IF (PR(N).LI.PRSVMX) GO TO 12
115000 PRSVMX=PR(N)
115100 NHOLD=N
115200 X=X+1./ZD
115300 Y=Y+1./ZC
115400 C
115500 C
115600 C
115700 C
115800 C
115900 C
116000 C
116100 H(N3)=MO*RX*(N3)
116200 IF (H(N3) -GE- HIN) PR(N3)=0.000
116300 IF (H(N3) -GE- HIN) H(N3)=HIN
116400 IF (H(N3).GT.HMIN) GO TO 17
116500 HMIN=H(N3)
116600 HMIN=HMIN
116700 NSAVE=N3
116800 CONTINUE
116900 PH(N3)=PR(N3)*(H(N3)+H1.5)
117000 CONTINUE
117100 CONTINUE
117200 IF (M .EQ. 1) GO TO 20
117300 WRITE(6,900) (PR(N),N=1,NXY)
117400 FORMAT(1H ,2MPR/70(1H ,10D13.5/))
117500 WRITE(6,1100) NSAVE,HMIN
117600 FORMAT(1H ,4HNSAVE=,110,10X,5HMIN=,D16.5)
117700 WRITE(6,1200) PRSVMX,NHOLD
117800 FORMAT(1H ,7HPRSVMX=,D16.5,10X,6HMHOLD=,110)
117900 CONTINUE
118000 WRITE(6,RADIUS)

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```

018100 1106 WRITE(6,1106) NX,NY,MZON2,JZON
018200 1106 FORMAT(IX,'NX=',I6,' NY=',I6,' MZON2=',I6,' JZON=',I6)
018300 C
018400 C
018500 C
018600 C
018700 C
018800 C
018900 C
019000 21
019100 C
019200 C
019300 C
019400 22
019500 C
019600 C
019700 C
019800 C
019900 C
020000 C
020100 C
020200 C
020300 C
020400 C
020500 C
020600 C
020700 C
020800 C
020900 C
021000 C
021100 C
021200 C
021300 C
021400 C

INITIAL VISCOSITY AND DENSITY CALCULATION
DO 21 N=1,NX
  DEN(N)=1.+(Q5*PR(N))/(1.+Q6*PR(N))
  VIS(N)=Q1*W(1.-(1.+Q2*PR(N))*W2)
  XMU(N)=CENS(N)/VIS(N)
CONTINUE

RELAXATION COEFFICIENTS A,B,C,D,L,AND H
CONTINUE
SUM=0.000
ZC=ZC1
ZF=ZF1
DO 24 J=2,NY
  DO 23 I=2,NX1
    ZD=ZD1
    ZE=ZE1
    M=I+(J-1)*NX
    NVMF=NCGF+(J-1)*MX
    NVMB=NVMF+NZON2
    IF(J.GE.JZON) ZF=ZC2
    IF(J.GT.JZON) ZC=ZC2
    IF(N.GE.NVMF .AND. N.LT.NVMB) ZE=ZE1
    IF(N.GT.NVMF .AND. N.LE.NVMB) ZD=ZE1
    IF(N.GE.NVMB) ZE=ZE2
    IF(N.GT.NVMB) ZD=ZE2
    V1=((ZE*ZD)/(ZE+ZD))*W2
    V2=((ZE)*W2.)*M(ZD-ZE)/(ZD+ZE)
    V3=((ZE*W3.)*M(ZE+ZD)/(ZE+ZD))*W2
    V4=((ZD*W3.)*M(2.*ZE+ZD)/(ZE+ZD))*W2.
  
```

```

21500 V5=(ZD/ZE)M2.MV2
21600 C2D=ZCM2-ZFM2
21700 C3P1=ZC+2.MZF
21800 C3P2=2.MZC+ZF
21900 CF=(ZF/(ZC+ZF))M2
22000 CF1=(1./(ZC+ZF))M2
22100 CP=(ZC+ZF)M2
22200 Z1=-5*ZD
22300 Z2=ZC/(ZC+ZF)
22400 Z3=Z1M2
22500 Z4=Z2M2
22600 M1=M+1
22700 M2=M-1
22800 M3=M+MX
22900 M4=M-MX
23000 IF (J.EQ.NY) M3=M4
23100 Y0=XMUC(M)
23200 Y1=XMUC(M1)
23300 Y2=XMUC(M2)
23400 Y3=XMUC(M3)
23500 Y4=XMUC(M4)
23600 Y5=M(M1)
23700 Y6=M(M2)
23800 Y7=M(M)
23900 Y8=M(M3)
24000 Y9=M(M4)
24100 Y10=Y1+DSQRT(Y5)
24200 Y11=Y2+DSQRT(Y6)
24300 Y12=Y3+DSQRT(Y8)
24400 Y13=Y4+DSQRT(Y9)
24500 Y14=Y0+DSQRT(Y7)
24600
24700 A(N)=V1M2+V2MY0+V3MY1
24800 B(N)=Z4M(Y3+ZFM2-C2D)MY0+ZCM3P1MY4)
24900 C(N)=V4MY2-V5MY0+V1MY1
25000 D(Z,M)=CFM(ZFMC3P2MY1+C2D)MY0+ZCM2MY4)
25100 X11=Y2M4.MZ3-(ZD-ZE)M2MY0+Y1M2EM2+CF1M(Y3+ZFM2MCP-C2D)M2MY0+ZCM2MCPMY4)
25200 X12=1.5/(Y7M1.5)
25300 X13=Y11M(V4MY6-Y7M4.MZ3+Y5M1)+Y14M(-V3MY6+(ZD-ZE)M2MY7+Y5MY2)+
Y10M(V1MY6-ZEM2MY7+V3MY5)
25400 X14=CF1M(Y12+ZFM2M(ZFMC3P2MY0-CPMY7+Y9M2CM2)+Y13M2CM2 -
1M(Y8+ZFM2-CPMY7+ZCM3P1MY9)+Y14M2DM(ZFM2MY8+C2D)MY7-ZCM2MY9))
25500 X1(M)=X11+X12M(X13+X14)
25600 XN1=12.MU/(Y7M1.5)
25700 XM2=((ZEM2.MDEN(M1)MYS/(ZD+ZE)+(ZD-ZE)MDENS(M)MY7-ZDM2.M
1Y6/(ZD+ZE))MDCOS(THETA))
25800 XM3=((Y8+ZFM2+DEN(M3)+C2D)MDENS(M)MY7-Y9M2CM2MDENS(M4))MDSIM(THETA)
25900 XM(M)=XM1M(XM2+Z2/ZCMXM3)
26000 CONTINUE
26100 CONTINUE
26200 SUM=0.000
26300
26400 RELAXATION FORMULA
26500
26600 DO 30 J=2,NY
26700
26800

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```

126900      MENDA=0
127000      MEND=0
127100      DO 29 I=2,NX1
127200      MN=I+(J-1)*NX
127300      MNA=MN+1
127400      MNB=MN-1
127500      MNC=MN+NX
127600      MND=MN-NX
127700      IF(J.EQ.NY) MNC=MND
127800      ZPR(MN)=PHI(MN)-ORF*(PHI(MN)+(XM(MN)+X(MC(MN)))/XL(MN))
127900      C(MN)*PHI(MNB)-DLZ(MN)*PHI(MNC))/XL(MN))
128000      CONTINUE
128100      IF (CH(MN) .GE. HIN) GO TO 127
128200      IF (ZPR(MN).LE.0.) GO TO 27
128300      STON=STON+DABS(ZPR(MN))
128400      SERR=SERR+DABS(ZPR(MN))-PHI(MN))
128500      GO TO 28
128600      ZPR(MN)=0.000
128700      MENDA=MENDA+1
128800      IF(MENDA.EQ.1)XMI(J)=XG(I)
128900      GO TO 28
129000      ZPR(MN)=0.000
129100      MEND=MEND+1
129200      IF (MEND.EQ. 1) XE(J)=XG(I)
129300      PHI(MN)=ZPR(MN)
129400      CONTINUE
129500      CONTINUE
129600      MAUR=MAUR+1
129700      CONTINUE
129800      IF (MNT.EQ.0) GO TO 66
129900      CONTINUE
130000      WRITE (8) (PR(I),I=1,NXY)
130100      WRITE (8) (NCL(I),I=1,NXY)
130200      REMIND 8
130300      CONTINUE
130400      IF (SERR.LT.DEL3) GO TO 311
130500      GO TO 25
130600      C
130700      C
130800      C
130900      SUM2=0.000
131000      WRITE (6,480) MAUR,SERR
131100      FORMAT(1H ,5HMAUR=,18,5X,4HSERR=,D16.5)
131200      MAUR=0
131300      PRMX= 10-13
131400      ROSM=ROSM1
131500      TDH=0.0
131600      ERK1=0.0
131700      SUM2=0.0
131800      DO 54 J=2,NHY
131900      JP=(J-1)*NX
132000      DO 54 I=2,NXM1
132100      N=I+JP
132200      A(N)=PHI(N)/(H(N)*M1.5)

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332300 DLZ(N)=A(N)-PR(N)
332400 TDN=TDN+DABS(A(N))
332500 ERR1=ERR1+DABS(DLZ(N))
332600 54 CONTINUE
332700 ERR=ERR1/TDN
332800 DO 315 J=2,NY
332900 DO 316 I=2,NX1
333000 N=I+(J-1)*NX
333100 PR(N)=(PHI(N)/(H(N)*W1.5)+(ROSM-1.)*WPR(N))/ROSM
333200 IF (H(N) -GE. MIN) PR(N)=0.000
333300 IF (PR(N).LT.PRMX) GO TO 33
333400 PRMX=PR(N)
333500 PRMXS=PRMX
333600 MHOLD=N
333700 33 CONTINUE
333800 C IF (NVISC.EQ.0) GO TO 316
333900 DENSN=1.+(Q5*WPR(N))/(1.+Q6*WPR(N))
334000 VISH=Q1*W(1.-(1.+Q2*WPR(N))*2)
334100 XMUN=DENSN/VISH
334200 Y99=(XMUN-XMUN(N))/XMUN
334300 SUM2=SUM2+DABS(Y99)
334400 DENSN(N)=DENSN
334500 VIS(N)=VISH
334600 XMUN(N)=XMUN
334700 315 CONTINUE
334800 MNI=1
334900 459 CONTINUE
335000 JENN=JENN+1
335100 WRITE(6,810) SUM2
335200 810 FORMAT(1H ,5HSUM2=,D16.5)
335300 IF (ERR.LT.DEL1) GO TO 32
335400 MAXS2=MAXS2+1
335500 GO TO 22
335600 C APPLIED NORMAL LOAD
335700 C
335800 C CONTINUE
335900 32 MAXS2=1
336000 WRITE(6,481) JENN
336100 481 FORMAT(1H ,9HJENNIFER=,I8)
336200 JENN=0
336300 MAUR=0
336400 QUA=0.000
336500 PSUM=0.000
336600 IZON=1
336700 ZD=ZD1
336800 PBAR=0.000
336900 PCHECK=0.000
337000 MS=2
337100 MF=HCGF
337200 GO TO 483
337300 483 ZD=ZE1
337400 IZON=2
337500 MS=MF
337600

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337700 MF=MS+NZON2
337800 QW=0.000
337900 PSUM=0.000
338000 GO TO 483
338100 485 ZD=ZE2
338200 IZON=3
338300 MS=MF
338400 MF=MX
338500 QW=0.000
338600 PSUM=0.000
338700 483 CONTINUE
338800 DO 38 I=MS,MF
338900 IM=MOD(I,2)
339000 IMS=MOD(MS,2)
339100 IF(IMS.EQ.1) GO TO 486
339200 QUI=1.000
339300 IF(IM.EQ.1) QUI=2.000
339400 GO TO 487
339500 486 QUI=2.000
339600 IF(IM.EQ.1) QUI=1.000
339700 487 CONTINUE
339800 QUS=1.000
339900 QUS=0.000
340000 IF(I.EQ.MS .OR. I.EQ.MF) QUI=-.500
340100 IF(I.EQ.MS .OR. I.EQ.MF) QUS=-.500
340200 JINTG=1
340300 ZC=ZC1
340400 NS=2
340500 MF=JZON
340600 GO TO 35
340700 34 JINTG=2
340800 ZC=ZC2
340900 MS=MF
341000 MF=MY
341100 35 DO 37 J=MS,MF
341200 M=1+(J-1)*MX
341300 JN=MOD(J,2)
341400 JNS=MOD(MS,2)
341500 IF(JNS.EQ.1) GO TO 40
341600 QVI=2.000
341700 IF(JN.EQ.1) QVI=4.000
341800 IF(J.EQ.MS .OR. J.EQ.MF) QVI=1.000
341900 GO TO 37
342000 40 QVI=4.000
342100 IF(JN.EQ.1) QVI=2.000
342200 IF(J.EQ.MS .OR. J.EQ.MF) QVI=1.000
342300 37 QUS=QUS+PR(M)*QVI/ZC
342400 IF(JINTG.EQ.1) GO TO 34
342500 PSUM=PSUM+QUS*QUS/3.0
342600 38 QUS=QUS+QUS*QUS/3.0
342700 PBAR=4.*QUS/(3.*ZD)+PBAR
342800 PCHECK=2.*PSUM/ZD+PCHECK
342900 FBAR=4.*MF/(RX*M2)*QUS/(3.*ZD)+FBAR
343000 FCHECK=2.*MF/(RX*M2)*PSUM/ZD+FCHECK

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043100 IF (IZON.EQ.2) GO TO 485
043200 IF (IZON.EQ.3) GO TO 36
043300 GO TO 482
043400 CONTINUE
043500 WOU=PBAR/(0.411D-SVVRX)
043600 WRITE (6,OUTPUT)
043700 WS=PBAR
043800 CONTINUE
043900 IF (MPARA.EQ.0) GO TO 43
044000 PRMX=LD-13
044100 PRSMX=PRMX
044200 HMAX=10.0D0
044300 HMIN=10.0D0
044400 MPARA=0
044500 GO TO 976
044600 CONTINUE
044700 IF (NVISC.EQ.0) GO TO 53
044800 WRITE(6,1600) NSAVE,HMIN
044900 FORMAT(1H,6HNSAVE=,I10,10X,5HMIN=,D16.5)
045000 WRITE(6,1700) (PR(N),N=1,NX)
045100 FORMAT(1H,2HPR/70(1H,10D13.5/))
045200 WRITE(6,1900) (H(N),N=1,NX)
045300 FORMAT(1H,1H/70(1H,10D13.5/))
045400 WRITE(6,1800) (PHI(N),N=1,NX)
045500 FORMAT(1H,3HPI/70(1H,10D13.5/))
045600 WRITE(6,2000) (S(N),N=1,NX)
045700 FORMAT(1H,1HS/70(1H,10D13.5/))
045800 WRITE(6,2100) (DENS(N),N=1,NX)
045900 FORMAT(1H,7HDENSITY/70(1H,10D13.5/))
046000 WRITE(6,2200) (VIS(N),N=1,NX)
046100 FORMAT(1H,9HVIScosity/70(1H,10D13.5/))
046200 C
046300 53
046400 CONTINUE
046500 PMIN=.1E-20
046600 PRMX=.1D-20
046700 DO 55 J=1,NY
046800 DO 54 I=1,NX
046900 N=I+NX*(J-1)
047000 JMIN=NY1-J
047100 PRS(N)=PR(N)
047200 PRD(I,J)=PRS(N)
047300 C FMIX(1,I,J)=XG(I)
047400 C FMIX(2,I,J)=YGC(J)
047500 C FMIX(3,I,J)=PRD(I,J)
047600 HS(N)=H(N)
047700 IF (PR(N).LT.PRMX) GO TO 54
047800 PRMX=PR(N)
047900 PRMS=PR(N)
048000 MHOLD=N
048100 XPK=XG(I)
048200 54
048300 55 CONTINUE
048400

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048500      NYMM1=NY-1
048600      DO 56 I=1,NX
048700      N=I+NX+NYMM1
048800      YG(I)=PR(N)
048900      CONTINUE
049000      WRITE(6,3100)(XMI(N),N=1,NY)
049100      FORMAT(1X,5HXMI= /70(1X,10D13.5/))
049200      HMAX=H(NVME+1)
049300      PHIN=PR(NVME+1)
049400      WRITE(6,3000) PRMX,NHOLD
049500      FORMAT(1H,5HPRMX=,D16.5,10X,6HNHOLD=,I10)
049600      WRITE(6,3200) VIS(NHOLD)
049700      FORMAT(1X,5HVIS(NHOLD)=,D16.5)
049800      WRITE(8) NX,NY,((PRD(I,J),I=1,NX),J=1,NY),(XG(N),N=1,NX),(YGC(N),N=1,NY),PRMXS
049900      NVMB=NCGF+NZONZ
050000      WRITE(8)NXY,NCGF,NVMB,JZON
050100      CONTINUE
050200      CALL DISPLAY(XE,NY,XPK,PCHECK,PBAR,PRMXS,NHOLD)
050300      CONTINUE
050400      STOP
050500      END
050600

```

APPENDIX 3

COMPUTER PROGRAM LIST 2

PROGRAM NAME
CHHR

UTILITY
Calculate the deformation under
contact stresses by the method
proposed by Hamrock and Dowson.

CNHR

Calculate the deformation under
contact stresses
by the present approach.

```

000020 C      CHNR
000100      IMPLICIT REAL*8(A-H,O-Z)
000200      REAL*4 XE(70)
000300      DIMENSION XD(150),YD(70),PR(3600)
000400      DIMENSION D(2000)
000500      DIMENSION W(2000),H(2000),S(2000)
000600      DIMENSION W(2000),VIS(2000),XMU(2000),PHI(2000)
000700      COMMON/AA/MD,PRX,HMIM,MHSAVE
000800      NAMELIST /INPUT/MD,SMU,SMV,EP,FP
000900      NAMELIST/OUTPUT/G,ALPHA,RBY,XK,PBAR,PCHECK,FBAR,FCHECK,STRS,NAMERK
001000 C      DATA PI/3.1415926535900/
001100 C      DATA RAX,RAY,RBX/1.1112500,1.1112500,1.0D12/
001200 C      ***
001300 C
001400 C      DATA RX/1.1112500/
001500 C      DATA RAY,RBY/1.1112500,-1.1861150D0/
001600 C      DATA XI,YI,MN/0.0,0.0,0.19/
001700 C      DATA PI,EP/3.1415926535900,21.97D06/
001800 C      DATA FP,EX,RV/8.964D0, 0.5558D0,0.5558D0/
001900 C      ***
002000 C      DATA ZD,ZC/13.0D0,5.0D0/
002100 C      DATA MX,MNY/39.20/
002200 C      DATA XM,YM/1.0D0,1.0D0/
002300 C      DATA MD/0.0/
002400 C
002500 C      MD=0.0
002600 C      RY=1.0/((1.0/RAY)+(1.0/RBY))
002700 C      NXHY=NX*MNY
002800 C      NY=2*MNY-1
002900 C      NXY=NX*MNY
003000 C      NXMI=NX-1
003100 C
003200 C      CURVATURE SUM AND DIFFERENCE
003300 C
003400 C      PRX=1.0/RX
003500 C      PRY=1.0/RY
003600 C      RHO=PRX*PRY
003700 C      GAMMA=(PRX-PRY)/RHO
003800 C      ALPHA=PRX/PRY
003900 C      PAL=1.0/ALPHA
004000 C      PDAL=PAL*PAL
004100 C
004200 C      INITIAL GUESS FROM SIMPLIFY EQ
004300 C      Q=PI/2.0-1.0
004400 C      FSS=PI/2.0+Q*MDLOG(ALPHA)
004500 C      ESS=1.0*Q/ALPHA
004600 C      XK=ALPHARK(2.0D0/PI)
004700 C
004800 C      ELLIPTIC INTEGRALS ES AND FS
004900 C      XK=SQRT(2.0D0)
005000 C      XK=XK
005100 C
005200 C

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0005300
0005400 C
0005500 C
0005600 C
0005700 C
0005800
0005900 1
0006000
0006100
0006200
0006300
0006400
0006500 C
0006600 C
0006700 C
0006800 C
0006900 C
0007000
0007100
0007200
0007300 C
0007400 C
0007500 C
0007600
0007700
0007800
0007900 1010
0008000 C
0008100 C
0008200
0008300
0008400
0008500
0008600
0008700
0008800
0008900 C
0009000 C
0009100 C
0009200
0009300
0009400
0009500
0009600
0009700
0009800
0009900
0010000
0010100
0010200 5
0010300
0010400
0010500 6
0010600

ES=ESS
FS=ESS
XK IS THE ELLIPTICITY PARAMETER
KITER=0
KITER=KITER+1
AK=SQRT(1.0D0-(1.0D0/(XK**2)))
CALL CEL(FS,ES,AK,IER)
XK=SQRT((2.0MF5-ES*(1.0+GAMMA))/(ES*(1.0-GAMMA)))
A=XK-XJK
X=XJK
IF(DABS(A9).GT.1.0D-8) GO TO 1
INITIAL SEMIMAJOR AND SEMIMINOR AXES OF CONTACT ELLIPSE
FP=WDNEP/(PRX*PRX)
WD=FP*PRX*PRX/EP
AXI=(6.0*XK*XK*ES*FP/(PI*EP*RH0))**(.9/3.0)
BXI=AXI/XK
PH2MX=1.5*FP/(EP*PI*AXI*BXI)
DDC=PRX*FS*(((4.5*RH0)/ES)*((FP/(PI*EP*MXK))**2)**((1.0/3.0)
DDC=FS*(((4.5*RH0)/ES)*((FP/(PI*EP*MXK))**2)**((1.0/3.0)
WRITE(6,1010) DDC,AXI,BXI
FORMAT(1X,'DDC=',D12.5,' AXI=',D12.5,' BXI=',D12.5)
PDXX=1.0/(XK*MXK)
PXX=1.0/XK
T3=0.5*PRX*BXI*MXI
T2=0.5*PRY*AXI*MAXI
Z1=0.5*ZD
Z2=0.5*ZC/XK
Z3=Z1*Z2
Z4=Z2*Z2
NXMI=NX-1
MODAL STRUCTURE
DZD= XM/MN
DZC= YM/MN
ZD=1.0/DZD
ZC=1.0/DZC
HDZD=0.5*HDZD
HDZC=0.5*HDZC
XD(1)=0.0
YD(1)=0.0
DO 5 I=2,NX
XD(I)=XD(I-1)+DZD
CONTINUE
DO 6 I=2,NY
YD(I)=YD(I-1)+DZC
CONTINUE
NCTR=NX*NY-NX*MN+1

```

```

1010700 C
1010800 C
1010900 C
1011000
1011100
1011200
1011300
1011400
1011500
1011600
1011700
1011800
1011900
1012000
1012100 C
1012200 C
1012300
1012400 7
1012500 C
1012600 C
1012700 9
1012800 8
1012900 C
1013000 C
1013100 C
1013200
1013300
1013400
1013500
1013600
1013700
1013800
1013900
1014000
1014100
1014200
1014300
1014400
1014500
1014600
1014700
1014800
1014900
1015000 85
1015100 65
1015200 C
1015300 C
1015400 C
1015500 10
1015600
1015700
1015800 C
1015900 C
1016000 3401

CURVATURE AND HERTZIAN PRESSURE

DO 8 J=1,NHY
JP=(J-1)*MX
DO 9 I=1,NX
N=I+JP
S20=(YD(J)-YM)*M2
S21=(XD(I)-XM)*M2
S51=S20+S21
S(N)=T2+S20+T3+S21
ECIR=1.0-SS1
IF (ECIR.LE.0.0) GO TO 7
PR(N)=PHZMX*ECIR*0.5
PRSV(N)=PR(N)
HCN=H0+DDC
GO TO 9
PR(N)=0.0D0
PRSV(N)=PR(N)
HCN=H0+S(N)*PRX
CONTINUE
CONTINUE

INFLUENCE COEFFICIENT

DXK=XK*XX
DO 65 JJ=1,NY
JP=(JJ-1)*MX
Y2=(JJ-1)*HDZC
CP=(Y2+HDZC)*XX
CM=(Y2-HDZC)*XX
DO 85 II=1,NX
N=II+JP
X2=(II-1)*HDZD
S1=DSQRT(DXK*(Y2+HDZC)*M2+(X2+HDZD)*M2)
S2=DSQRT(DXK*(Y2-HDZC)*M2+(X2-HDZD)*M2)
S3=DSQRT(DXK*(Y2+HDZC)*M2+(X2-HDZD)*M2)
S4=DSQRT(DXK*(Y2-HDZC)*M2+(X2-HDZD)*M2)
DP=X2+HDZD
DM=X2-HDZD
D(N)=(DP*DLG((CP+S1)/(CM+S2))+CP*DLG((DP+S1)/(DM+S3))+
DM*DLG((CM+S4)/(CP+S3))+CM*DLG((DM+S4)/(DP+S2)))*M2)*M2)

CONTINUE
CONTINUE

INITIAL FILM THICKNESS VISCOSITY AND DENSITY CALCULATION

CONTINUE
CALL SM2(MX,NY,NHY,NXY,NXHY,XD,YD,D.H,M,S,PR,PHI)
NHTH=DDC*H0
WRITE(6,3401) HMIN,NHSAVE,H(NCTR),H(NCTR)
WRITE(6,3401) HMIN,NHSAVE,H(NCTR),H(NCTR)
FORMAT(3X,' HMIN=',D14.5,' NHSAVE=',I6,' H(NCTR)='',D14.5,' H(NCTR)='',D14.5)

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ORIGINAL PAGE IS
OF POOR QUALITY

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3016100 312 CONTINUE
3016200 DLW=H(NCTR)/DDC-1.0
3016220 DLW1=(W(NCTR)+H(NCTR-1))/(2.0*DDC)-1.0
3016300 DLH=H(NCTR)-HMIN
3016320 DLH1=(H(NCTR)+H(NCTR-1))/2.0-HMIN
3016400 WRITE (6,300) NX,NY,NHY,MN,FP
3016500 WRITE (4,300) NX,NY,NHY,MN,FP
3016600 300 FORMAT(IX,'NX=',I5,' NY=',I5,' NHY=',I5,' MN=',I5,' FP=',D10.5)
3016700 WRITE (6,250) ZD,ZC,XI,YI
3016800 WRITE (4,250) ZD,ZC,XI,YI
3016900 250 FORMAT(IX,'ZD=',D10.4,' ZC=',D10.4,' XI=',D10.4,' YI=',D10.4,IX)
3017000 WRITE (6,500) PHZMX,DLW,DLH,RX,RX,R
3017100 WRITE (4,500) PHZMX,DLW,DLH,RX,RX,R
3017200 500 FORMAT(IX,'PHZMX=',D11.5,' DLW=',D11.5,' DLH=',D11.5,' RX=',
D11.5,' RY=',D11.5,' R=',D11.5)
3017220 WRITE(6,600) DLW1,DLH1
3017225 WRITE(4,600) DLW1,DLH1
3017240 600 FORMAT(IX,'DLW1=',D14.5,' DLH1=',D14.5)
3017300 DO 50 JW=1,NHY
3017400 IWP=(JW-1)*NX
3017500 IE=IWP+1
3017600 IS=IWP+NX
3017700 PRINT 150,JW
3017800 PRINT 200,(W(M),M=IE,IS)
3017900 PRINT 200,(H(M),M=IE,IS)
3018000 WRITE(6,150) JW
3018100 WRITE(6,200)(W(M),M=IE,IS)
3018200 WRITE(6,200)(H(M),M=IE,IS)
3018300 CONTINUE
3018400 50 FORMAT(/SX,'JW=',I5)
3018500 150 FORMAT(/,10(IX,D12.5))
3018600 200 STOP
3018700 END

```

```

000020 C
000100 C      CNMR      IMPLICIT REAL*8(A-H,O-Z)
000300      DIMENSION XD(150),YD(70),PR(3600)
000400      DIMENSION D(39,39,9)
000500      DIMENSION W(2000),H(2000),S(2000)
000600      DIMENSION DENS(2000),VIS(2000),XHU(2000),PHI(2000)
000620      COMMON B222,B322,B422,B522,B212,B312,B232,B332,B423,B523,B421,B521
000640      COMMON/AA/HO,PRX,RHIM,MHSAVE
000800      NAMELIST /INPUT/HO,SMU,SMV,EP,FP
000900      NAMELIST/OUTPUT/G,ALPHA,RBY,XK,PBAR,PCHECK,FBAR,FCHECK,STRS,NAMERK
001000      DATA PI/5.141592653590/
001100 C      DATA RAX,RAY,RBX/1.1112500,1.1112500,1.1112500,1.0012/
001200 C      ***      ***
001300 C
001400 C      DATA RX/1.1112500/
001500 C      DATA RAY,RBY/1.1112500,-1.186115000/
001600 C      DATA XI,YI,MH/0.0,0.19/
001700 C      DATA PI,EP/3.1415926535900,21.97006/
001800 C      DATA FP,RX,RX/8.96400, 0.555800,0.555800/
001900 C      ***
002000 C      DATA ZD,ZC/13.000,5.000/
002100 C      DATA NX,MHY/39,20/
002200 C      DATA XM,YM/1.000,1.000/
002300 C      DATA HO/0.0/
002400 C
002500 C      HO=0.0
002600 C      RY=1.0/((1.0/RAY)*(1.0/RBY))
002700 C      HXHY=NX*MHY
002800 C      NY=2*MHY-1
002900 C      HXY=NX*MHY
003000 C      NXMI=NX-1
003100 C
003200 C      CURVATURE SUM AND DIFFERENCE
003300 C
003400 C      PRX=1.0/RX
003500 C      PRY=1.0/RBY
003600 C      RHO=PRX*PRY
003700 C      GAMMA=(PRX-PRY)/RHO
003800 C      ALPHA=PRX/PRY
003900 C      PAL=1.0/ALPHA
004000 C      PDAL=PAL*PAL
004100 C
004200 C      INITIAL GUESS FROM SIMPLIFY EQ
004300 C
004400 C      Q=PI/2.0-1.0
004500 C      ESS=PI/2.0+RMDLOG(ALPHA)
004600 C      ESS=1.0*Q/ALPHA
004700 C      XKK=ALPHAK*(2.000/PI)
004800 C
004900 C      ELLIPTIC INTEGRALS ES AND FS
005000 C
005100 C      XK=SQRT(2.000)
005200 C      XK=XKK

```


ORIGINAL PAGE IS
OF POOR QUALITY

```

305300 ES=ESS
305400 FS=FSS
305500 C
305600 C
305700 C
305800 C
305900 1
306000 KITER=KITER+1
306100 AK=SQRT(1.0D0-(1.0D0/(XK*W2)))
306200 CALL CEL(FS,ES,AK,IER)
306300 XJK=SQRT((2.0*FS-ES*(1.0+GAMMA))/(ES*(1.0-GAMMA)))
306400 A9=XK-XJK
306500 XK=XJK
306600 IF(DABS(A9).GT.1.0D-8) GO TO 1
306700 C
306800 C
306900 C
307000 FP=WDWEP/(PRX*PRX)
307100 WD=FP*PRX*PRX/EP
307200 AX1=(6.0*XK*XK*ES*FP/(PI*EP*RH0))**W2*(1.0/3.0)
307300 C
307400 C
307500 C
307600 C
307700 C
307800 C
307900 1010
308000 C
308100 C
308200 C
308300 C
308400 C
308500 C
308600 C
308700 C
308800 C
308900 C
309000 C
309100 C
309200 C
309300 C
309400 C
309500 C
309510 C
309520 C
309530 C
309540 C
309600 C
309700 C
309800 C
309900 C
310000 C
310100 5
310200 C

XK IS THE ELLIPTICITY PARAMETER

KITER=0
KITER=KITER+1
AK=SQRT(1.0D0-(1.0D0/(XK*W2)))
CALL CEL(FS,ES,AK,IER)
XJK=SQRT((2.0*FS-ES*(1.0+GAMMA))/(ES*(1.0-GAMMA)))
A9=XK-XJK
XK=XJK
IF(DABS(A9).GT.1.0D-8) GO TO 1

INITIAL SEMIMAJOR AND SEMIMINOR AXES OF CONTACT ELLIPSE

FP=WDWEP/(PRX*PRX)
WD=FP*PRX*PRX/EP
AX1=(6.0*XK*XK*ES*FP/(PI*EP*RH0))**W2*(1.0/3.0)
BX1=AX1/XK

PH2MX=1.5*FP/(EP*PI*AX1*BX1)
DDC=PRX*FS*((4.5*RH0)/ES)*H(FP/(PI*EP*MX))**W2*(1.0/3.0)
DDC=FS*((4.5*RH0)/ES)*H(FP/(PI*EP*MX))**W2*(1.0/3.0)
WRITE(6,1010) DDC,AX1,BX1
WRITE(6,1010) DDC,AX1,BX1
FORMAT(1X,DDC=' ',D12.5,' AX1=' ',D12.5,' BX1=' ',D12.5)
PDXK=1.0/(XK*XK)
PXX=1.0/XK
T3=0.5*PRX*BX1*MX1
T2=0.5*PRX*AX1*MX1
Z1=0.5*ZD
Z2=0.5*ZC/XK
Z3=Z1*Z2
Z4=Z2*Z2
MXM1=MX-1

MODAL STRUCTURE

DZD= XM/MH
DZC= YM/MH
ZD=1.0/DZD
ZC=1.0/DZC
RDZD=BX1*HDZD
RDZC=AX1*HDZC
RZD=1.0/RDZD
RZC=1.0/RDZC
HDZD=0.5*HDZD
HDZC=0.5*HDZC
XD(1)=0.0
YD(1)=0.0
DO 5 I=2,NX
XD(I)=XD(I-1)+DZD
CONTINUE

```


ORIGINAL PAGE IS
OF POOR QUALITY

```

013460      B222=-(X3+X1)
013480      B322=X3+X1
013500      B212=-(X3+X2)
013520      B312=X3+X2
013540      B232=-(X1+X2)
013560      B332=X1+X2
013580      D(11,JJ,5)=B122*(F22(X1,Y1)+F22(X3,Y3)-F22(X1,Y3)-F22(X3,Y1))
013600      D(11,JJ,4)=B112*(F12(X1,Y1)+F12(X3,Y3)-F12(X1,Y3)-F12(X3,Y1))
013620      D(11,JJ,6)=B132*(F32(X1,Y1)+F32(X3,Y3)-F32(X1,Y3)-F32(X3,Y1))
013640      D(11,JJ,8)=B123*(F23(X1,Y1)+F23(X3,Y3)-F23(X1,Y3)-F23(X3,Y1))
013660      D(11,JJ,7)=B113*(F13(X1,Y1)+F13(X3,Y3)-F13(X1,Y3)-F13(X3,Y1))
013680      D(11,JJ,9)=B133*(F33(X1,Y1)+F33(X3,Y3)-F33(X1,Y3)-F33(X3,Y1))
013700      D(11,JJ,2)=B121*(F21(X1,Y1)+F21(X3,Y3)-F21(X1,Y3)-F21(X3,Y1))
013720      D(11,JJ,1)=B111*(F11(X1,Y1)+F11(X3,Y3)-F11(X1,Y3)-F11(X3,Y1))
013740      D(11,JJ,3)=B131*(F31(X1,Y1)+F31(X3,Y3)-F31(X1,Y3)-F31(X3,Y1))
013760      CONTINUE
013780      85
013800      65
015200      C
015300      C
015400      C
015500      10
015600
015700      CONTINUE
015800      CALL SAI(MX,NY,NMY,NXY,XD,YD,D,S,M,S,PR,PHI)
015900      HNTN=DDC+MO
015900      WRITE(6,3401) HMIN,NHSAVE,M(NCTR),M(NCTR)
015900      WRITE(6,3481) HMIN,NHSAVE,M(NCTR),M(NCTR)
016000      3481
016100      312
016200      CONTINUE
016300      DLN=M(NCTR)/DDC-1.0
016400      DLN=H(NCTR)-HMIN
016500      WRITE(6,300) MX,NY,NMY,NM,FP
016600      WRITE(6,300) MX,NY,NMY,NM,FP
016700      FORMAT(1X,NX=,15, NY=,15, NMY=,15, NM=,15, FP=,D10.5)
016800      WRITE(6,250) 2D,ZC,XI,YI
016900      250
017000      FORMAT(1X,ZD=,D10.4, ZC=,D10.4, XI=,D10.4, YI=,D10.4,1X)
017100      WRITE(6,500) PHZX,DLN,DLH,RX,RV,R
017200      500
017300      FORMAT(1X,PHZX=,D11.5, DLN=,D11.5, DLH=,D11.5, RX=,D11.5,
017400      RV=,D11.5, R=,D11.5)
017500      DO 50 JH=1,NMY
017600      JH=(JH-1)*NX
017700      IE=IMP+1
017800      IS=IMP+NX
017900      PRINT 150,JH
018000      PRINT 200,(M(M),M=IE,IS)
018100      PRINT 200,(M(M),M=IE,IS)
018200      WRITE(6,150) JH
018300      150
018400      WRITE(6,200) (M(M),M=IE,IS)
018500      200
018600      CONTINUE
018700      FORMAT(1X, JH=,15)
018800      FORMAT(1X,10(1X,D12.5))
018900      STOP
019000      END

```

APPENDIX 4

COMPUTER PROGRAM LIST 3

PROGRAM NAME
SEHLI

UTILITY
Analyze the surface roughness
effect on the EHL elliptical contacts.

SAI

Subprogram used to calculate the
deformation under contact stresses.

```

00020 C
00100 SENLI
00200 IMPLICIT REAL*8(A-M,O-Z)
00300 REAL*4 XE(70)
00400 DIMENSION XC(150),YD(70),PR(5000)
00500 DIMENSION FFI(1800),FF22(1800)
00600 DIMENSION D(69,17,9)
00700 DIMENSION WC(1600),H(1600),S(1600),GPR(3200)
00800 DIMENSION DENS(1800),VIS(1800),XMU(1800),PHI(1800)
00900 DIMENSION A(1800),B(1800),C(1800),DLZ(1800),XL(1800),XM(1800)
01000 DIMENSION PRLD(1800),DVLDC(1800)
01100 DIMENSION ZPR(1800),PRSV(1800)
01200 COMMON B222,B322,B422,B522,B212,B312,B232,B332,B423,B523,B421,B521
01300 COMMON/AA/HO,PRX,HMIN,MHSAVE
01400 COMMON/BB/CF11,CF22
01500 NAMELIST /INPUT/HO,SMU,SMV,EP,FP,XLMD,GMM
01600 NAMELIST /OUTPUT/G,ALPHA,RBY,XK,PBAR,PCHECK,FBAR,FCHECK,STRS,HAMRK
01700 NAMELIST /FRIC/CFRI,CFRICK
01800 DATA PI/3.14159265359D0/
01900 DATA PIVAS,VISO,VISE/4858.7D0,4.11D-6,6.31D-9/
02000 DATA ELPHAB,ETA/5.82744D-6,1.68348D-5/
02100 DATA Z/0.67D0/
02200 DATA ZZ/0.13/
02300 DATA PHIMX/100.0D-10/
02400 *** *** ***
02500 DATA ROSM1,ROSM2,ROSM3,ROSM4,ROSM5/50.0,5.0,5.5,5.0,70.0/
02600 DATA ROSM/100.0/
02700 DATA DERR,ITER,TAU,ITSOR/500.0,0.0,0.0,0.0/
02800 DATA ORF/1.0D0/
02900 DATA DEL1,DEL2,DEL3/4.0D-4,0.034D-2,0.3D-3/
03000 DATA DDEL,DDEL1/0.025D0,0.09D0/
03100 DATA ITHAX,NFITMX,INTAU/138.40,0/
03200 C
03300 C
03400 C
03500 DATA RX/1.11125D0/
03600 DATA RY,RBY/1.11125D0,-1.186115D00/
03700 C
03800 DATA ZD,ZC/13.0D0,5.0D0/
03900 DATA MX,MHY/69.9/
04000 DATA XH,YH/4.0D0,1.6D0/
04100 C
04200 READ(5,INPUT)
04300 RY=1.0/((1.0/RBY)+(1.0/RBY))
04400 NXNY=NXMHY
04500 NY=2*MHY-1
04600 NXY=NXNY
04700 NXMI=NX-1
04800 HSMIN=6.4237D-6NRX
04900 SGM=HSMIN/XLMD
05000 CF11=3.0*(GMM-2.0)/(GMM+1.0)*SGM*SGM
05100 CF22=3.0*(1.0-2.0*GMM)/(1.0+GMM)*SGM*SGM
05200 CFDP=(3.0/(GMM+1.0))*SGM*SGM
05300 C
05400 C
05500 C
CURVATURE SUM AND DIFFERENCE

```

```

305600 C
305500
305500
305600
305700
305800
305900
306000
306100
306200 C
306300 C
306400 C
306500
306600
306700
306800
306900 C
307000 C
307100 C
307200 C
307300
307400
307500
307600 C
307700 C
307800 C
307900
308000 1
308100
308200
308300
308400
308500
308600
308700 C
308800 C
308900 C
309000 C
309100
309200
309300
309400 C
309500
309600
309700
309800
309900
310000 C
310100
310200 C
310300 C
310400 C
310500
310600
310700

PRX=1./RX
PRY=1./RY
RHO=PRX+PRY
GAMMA=(PRX-PRY)/RHO
ALPHA=PRX/PRY
PAL=1./ALPHA
PDAL=PAL/PAL
INITIAL GUESS FROM SIMPLIFY EQ
Q=PI/2.0-1.0
FSS=PI/2.0+Q*NDLOG(ALPHA)
ESS=1.0+Q/ALPHA
XKK=ALPHA*(2.000/PI)
ELLIPTIC INTEGRALS ES AND FS
XK=SQRT(2.000)
XK=XKK
ES=ESS
FS=FSS
XK IS THE ELLIPTICITY PARAMETER
KITER=0
KITER=KITER+1
AK=SQRT(1.000-(1.000/(XK**2)))
CALL GEL(FS,ES,AK,IER)
XJK=SQRT((2.0*FS-ES*(1.0+GAMMA)))/(ES*(1.0-GAMMA)))
AY=XK-XJK
XK=XJK
IF(DABS(AY).GT.1.0D-8) GO TO 1
INITIAL SEMIMAJOR AND SEMIMINOR AXES OF CONTACT ELLIPSE
FP=WDHEP/(PRX+PRY)
WD=FP*PRX+PRX/FP
AX1=(6.0*XK*XK*ES*FP/(PI*E*PHO))**0.5
BX1=AX1/XK
PHZMX=1.5*FP/(E*PI*MAX1*B*BX1)
DDC=PRX*FSS*((4.5*PHO)/ES)*M(FP/(PI*E*PHO))**0.5
WRITE(6,1010) DDC,AX1,BX1
WRITE(4,1010) DDC,AX1,BX1
FORMAT(1X,DDC=' ',D12.5,' AX1=' ',D12.5,' BX1=' ',D12.5)
PDXK=1.0/(XK*XK)
PAK=1.0/XK
MODAL STRUCTURE
DZD=1.0/ZD
DZC=1.0/ZC
RDZD=BX1*RDZD

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ORIGINAL PAGE IS
DE POOR QUALITY

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010800 RDZC=AX1WDZC
010900 RZD=1.0/RDZD
011000 RZC=1.0/RDZC
011100 HDZD=0.5WDZD
011200 HDZC=0.5WDZC
011300 XD(1)=HDZD
011400 YD(1)=0.0
011500 C RYD(1)=XD(1)*BX1
011600 C RYD(1)=YD(1)*MX1
011700 DO 5 I=2,NX
011800 XD(I)=XD(I-1)+DZD
011900 C RYD(I)=XD(I)*BX1
012000 5 CONTINUE
012100 DO 6 I=2,NY
012200 YD(I)=YD(I-1)+DZC
012300 C RYD(I)=YD(I)*MX1
012400 6 CONTINUE
012500 T3=0.5*PRX*MX1*BX1
012600 T2=0.5*PRY*MX1*MX1
012700 Z1=0.5*ZD
012800 Z2=0.5*ZC/XK
012900 Z3=Z1*WZ1
013000 Z4=Z2*WZ2
013100 C
013200 C
013300 C
013400 V=SQRT(SMUN*2+SHV*WZ2)
013500 THETA=ATAN(SMV/SMU)
013600 U=VISO*PRX*W/EP
013700 PHEE=1/(1+(2.*PRY)/(3.*PRX))
013800 Q1=VISE/VISO
013900 Q2=EP/19608.5268
014000 Q5=ELPHAMEP
014100 Q6=BETA*MEP
014200 G=EP/PI*VAS
014300 NCGF1=52
014400 NCTR=NX*HY-NX+NCGF1
014500 WRITE(6,INPUT)
014600 C
014700 C CURVATURE AND HERTZIAN PRESSURE
014800 C
014900 DO 8 J=1,NHY
015000 JP=(J-1)*MX
015100 DO 9 I=1,NX
015200 N=I+JP
015300 S20=(YD(J)-YM)*WZ2
015400 S21=(XD(I)-XM)*WZ2
015500 S51=S20+S21
015600 S(N)=T2*S20+T3*S21
015700 ECIR=1.0-S51
015803 IF (ECIR.LE.0.0) GO TO 7
015900 PR(N)=PHZ*MECIR*WZ0.5
016000 PRSV(N)=PR(N)
016100 GO TO 9

```

```

016200 7
016300
016400 9
016500 8
016600 C
016700 C
016800 C
016900
017000
017100
017200
017300
017400
017500
017600
017700
017800
017900
018000
018100
018200
018300
018400
018500
018600
018700
018800
018900
019000
019100
019200
019300
019400
019500
019600
019700
019800
019900
020000
020100
020200
020300
020400
020500
020600
020700
020800
020900 85
021000 65
021100
021200
021300
021400 C
021500

PR(N)=0.3D0
PRSV(N)=PR(N)
CONTINUE
CONTINUE

INFLUENCE COEFFICIENT

RZE=RZD
RZDE=RZD/2.0
B112=RZDHRZCHZC
B112=-RZDHRZCHZC
B132=-RZDHRZCHZC
B132=-RZDHRZCHZC
B133=-R112/2.0
B133=-R112/2.0
B133=-R132/2.0
B121=B123
B111=B113
B131=B133
DO 65 JJ=1,NY
Y2=(JJ-1)*RDZC
Y1=Y2+RDZC
Y3=Y2+RDZC
B422=-(Y3+Y1)
B522=Y3*Y1
B423=-(Y2+Y1)
B523=Y2*Y1
B421=-(Y2+Y3)
B521=Y2*Y3
DO 65 II=1,NX
X2=(II-1)*RDZD
X1=X2+RDZD
X3=X2+RDZD
B222=-(X3+X1)
B322=X3*Y1
B212=-(X3+X2)
B312=X3*Y2
B232=-(X1+X2)
B332=X1*Y2
D(II,JJ,5)=B122*(F22(X1,Y1)+F22(X3,Y3)-F22(X1,Y3)-F22(X3,Y1))
D(II,JJ,4)=B112*(F12(X1,Y1)+F12(X3,Y3)-F12(X1,Y3)-F12(X3,Y1))
D(II,JJ,6)=B132*(F32(X1,Y1)+F32(X3,Y3)-F32(X1,Y3)-F32(X3,Y1))
D(II,JJ,8)=B123*(F23(X1,Y1)+F23(X3,Y3)-F23(X1,Y3)-F23(X3,Y1))
D(II,JJ,7)=B113*(F13(X1,Y1)+F13(X3,Y3)-F13(X1,Y3)-F13(X3,Y1))
D(II,JJ,9)=B133*(F33(X1,Y1)+F33(X3,Y3)-F33(X1,Y3)-F33(X3,Y1))
D(II,JJ,2)=B121*(F21(X1,Y1)+F21(X3,Y3)-F21(X1,Y3)-F21(X3,Y1))
D(II,JJ,1)=B111*(F11(X1,Y1)+F11(X3,Y3)-F11(X1,Y3)-F11(X3,Y1))
D(II,JJ,3)=B131*(F31(X1,Y1)+F31(X3,Y3)-F31(X1,Y3)-F31(X3,Y1))
CONTINUE
CONTINUE
IF(MST.NE.1) GO TO 10
READ (9) (PR(N),N=1,NXHY)
READ (9) H001,(H(N),N=1,NXHY)
GO TO 99
DO 316 I=1,NXHY

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ORIGINAL PAGE IS
OF POOR QUALITY

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J21600 PR(I)=PR(I)*SQRT(H0/H001)
J21700 PRSV(I)=PR(I)
J21800 H(I)=H0-H001+H(I)
J21900 C
J22000 C
J22100 C
J22200 C
J22300 C
J22400 C
J22500 C
J22600 C
J22700 C
J22800 C
J22900 C
J23000 C
J23100 C
J23200 C
J23300 C
J23400 C
J23500 C
J23600 C
J23700 C
J23800 C
J23900 C
J24000 C
J24100 C
J24200 C
J24300 C
J24400 C
J24500 C
J24600 C
J24700 C
J24800 C
J24900 C
J25000 C
J25100 C
J25200 C
J25300 C
J25400 C
J25500 C
J25600 C
J25700 C
J25800 C
J25900 C
J26000 C
J26100 C
J26200 C
J26300 C
J26400 C
J26500 C
J26600 C
J26700 C
J26800 C
J26900 C
J27000 C
J27100 C
J27200 C

INITIAL FILM THICKNESS VISCOSITY AND DENSITY CALCULATION

CONTINUE
CALL SAI(MX,NY,NHY,NXY,XD,YD,D,H,W,S,PR,PHI,FF11,FF22)
FORMAT(3X,' HMIN=',D14.5,' NMSAVE=',I6,' HNRH=',D14.5,' H(NCTR)=' ,D14.5)
DO 21 N=1,NHY
  DENS(N)=1.+(Q5*PR(N))/(1.+Q6*PR(N))
  DENS(N)=1.0
  VIS(N)=Q1*W(1.-(1.+Q2*PR(N))*W2)
  XMU(N)=DENS(N)/VIS(N)
CONTINUE

RELAXATION COEFFICIENTS A,B,C,D,L,AND M

NITER=0
NITER=NITER+1
DEAR=5.0
ITER=0
ITER=ITER+1
DO 24 J=2,NHY
  JP=(J-1)*MX
  DO 23 I=2,NXMI
    R=I+JP
    N1=N+1
    N2=N-1
    N3=N+NX
    N4=N-NX
    IF (J.EQ.NHY) N3=N4
    Y1=XMU(N1)*FF11(N1)
    Y2=XMU(N2)*FF11(N2)
    Y3=XMU(N3)*FF22(N3)
    Y4=XMU(N4)*FF22(N4)
    Y5=H(N1)
    Y6=H(N2)
    Y7=H(N3)
    Y8=H(N4)
    Y9=H(N3)
    Y10=Y1*DSORT(Y5)
    Y11=Y2*DSORT(Y6)
    Y12=Y3*DSORT(Y8)
    Y13=Y4*DSORT(Y9)
    A(N)=Z3*H(3.*Y1+Y2)
    B(N)=Z4*H(Y3+3.*Y4)
    C(N)=Z3*H(Y1+3.*Y2)
    DLZ(N)=Z4*H(3.*Y3+Y4)
    XL1=4.*H(Z3*H(Y1+Y2)+Z4*H(Y3+Y4))
    XL2=1.5/(Y7*H1.5)
    XL3=Z3*H(Y10*H(3.*Y5-4.*Y7+Y6))+Y11*H(Y5-4.*Y7+3.*Y6))
    XL4=Z4*H(Y12*H(3.*Y8-4.*Y7+Y9))+Y13*H(Y8-4.*Y7+3.*Y9))
    XL(N)=XL1+XL2*H(XL3+XL4)
  
```

```

327300 XN1=12.0*XB1*PRX/(Y7*MI.5)
327400 XN2=(Y5*DENS(N1)-Y6*DENS(N2))*COS(THETA)
327500 XN3=(Y8*DENS(N3)-Y9*DENS(N4))*SIN(THETA)
327600 XN(N)=XN1*(Z1*XN2+Z2*XN3)
327700 CONTINUE
327800 23
327900 C
328000 C
328100 C
328200 20
328300 C
328400 25
328500 C
328600 C
328700 C
328800 C
328900 C
329000 C
329100 C
329200 C
329300 C
329400 C
329500 C
329600 C
329700 C
329800 C
329900 C

CONTINUE
ITSOR=0
ITSOR=ITSOR+1
STDM=0.0
SERR=0.0
SUM1=0.0
MSUM9=0
DO 30 J=2,NHY
  JP=(J-1)*NX
  MEND=0
  DO 29 I=2,NXMI
    MN=I+JP
    MNA=MN+1
    MNB=MN-1
    MNC=MN+NX
    MND=MN-NX
    IF(J.EQ.NHY) MNC=MND
    ZPR(MN)=PHI(MN)-ORF*(PHI(MN)+(XN(MN)-A(MN))*PHI(MNA)-B(MN))*PHI(MND)
    -C(MN)*PHI(MNB)-DLZ(MN)*PHI(MNC))/XL(MN))
    IF(ZPR(MN).LT.PHIMX) GO TO 26
    MSUM9=MSUM9+1
    ZPR(MN)=PHIMX
  CONTINUE
DO 26
  IF(ZPR(MN).LE.0.) GO TO 27
  Y18=(ZPR(MN)-PHI(MN))/ZPR(MN)
  SUM1=SUM1+DABS(Y18)
  STDM=STDM+DABS(ZPR(MN))
  SERR=SERR+DABS(ZPR(MN))-PHI(MN))
  GO TO 28
ZPR(MN)=0.000
STDM=STDM+DABS(ZPR(MN))
SERR=SERR+DABS(ZPR(MN))-PHI(MN))
MEND=MEND+1
IF(MEND.EQ.1) XE(J)=XD(I)
PHI(MN)=ZPR(MN)
CONTINUE
SERR=SERR/STDM
IF(SERR.GT.DELL) GO TO 25
IF(SUM1.GT.DELL) GO TO 25
CONTINUE

FILM THICKNESS VISCOSITY AND DENSITY ITERATION
CONTINUE
PRMX=.1D-13

```

ORIGINAL PAGE 12
OF POOR QUALITY

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032700 ROSM=ROSM1
032800 TDM=0.0
032900 ERR1=0.0
033000 SUM2=0.0
033100 DO 54 J=2,NHY
033200 JP=(J-1)*NX
033300 DO 54 I=2,NXM1
033400 N=I+JP
033500 A(N)=(PHI(N)/(H(N)*1.5)
033600 DLZ(N)=A(N)-PR(N)
033700 TDM=TDM+DABS(A(N))
033800 ERR1=ERR1+DABS(DLZ(N))
033900 CONTINUE
034000 ERR=ERR1/TDM
034100 WRITE(4,3000) ITER,ERR,OERR,INTAU
034200 WRITE(6,3000) ITER,ERR,OERR,INTAU
034300 FORMAT(1X,'ITER=',15,' ERR=',D14.5,' OERR=',D14.5,' INTAU=',15)
034400 IF(ERR.LE.0.05) ROSM=ROSM2
034500 IF(ERR.GE.0.5) ROSM=ROSM3
034600 IF(ERR.GE.1.0) ROSM=ROSM4
034700 IF(INTAU.LT.3.AND.ERR.GE.1.125*OERR) GO TO 411
034800 OERR=ERR
034900 OROSM=ROSM
035000 INTAU=0
035100 DO 416 J=2,NHY
035200 JP=(J-1)*NX
035300 DO 415 I=2,NXM1
035400 N=I+JP
035500 DVLD(N)=DLZ(N)
035600 PRLD(N)=PR(N)
035700 CONTINUE
035800 C414
035900 GO TO 312
036000 INTAU=INTAU+1
036100 ROSM=OROSM*2.0
036200 IF(ERR.GE.3.0*OERR) ROSM=OROSM*4.0
036300 IF(ERR.GE.5.0*OERR) ROSM=OROSM*6.0
036400 OROSM=ROSM
036500 DO 416 J=2,NHY
036600 JP=(J-1)*NX
036700 DO 417 I=2,NXM1
036800 N=I+JP
036900 DLZ(N)=DVLD(N)
037000 PR(N)=PRLD(N)
037100 CONTINUE
037200 C416
037300 CONTINUE
037400 DO 315 J=2,NHY
037500 JP=(J-1)*NX
037600 NEND=0
037700 DO 316 I=2,NXM1
037800 N=I+JP
037900 PR(N)=(PHI(N)/(H(N)*1.5)+(ROSM-1.0)*PRSV(N))/ROSM
038000 PRSV(N)=PR(N)

```

```

038100 IF(PR(N).LT.PRMX) GO TO 33
038200 PRM=PR(N)
038300 MPSAVE=N
038400 CONTINUE
038500 IF(PR(N).GT.0.1278000) PR(N)=0.1278000
038600 DENSM=1.+(05*PR(N))/(1.+06*PR(N))
038700 DENSM=1.0
038800 VISM=Q1M*(1.-(1.+02*PR(N))*M2)
038900 XMUN=DENSM/VISM
039000 Y99=(XMUN-XMUN(N))/XMUN
039100 SUM2=SUM2+DABS(Y99)
039200 DEN(N)=DENSM
039300 VIS(N)=VISM
039400 XMU(N)=XMUN
039500 CONTINUE
039600 VSMX=Q1M*(1.-(1.+02*PRM)*M2)
039700 WRITE (4,480) ITSOR,SERR,SUM1
039800 WRITE (6,480) ITSOR,SERR,SUM1
039900 FORMAT(5X,'ITSOR=',10,0X,'SERR=',D14.5,'SUM1=',D14.5)
040000 WRITE(6,3200) ROSM,PRM,VSMX,MPSAVE
040100 WRITE(6,3200) ROSM,PRM,VSMX,MPSAVE
040200 FORMAT(5X,'ROSM=',D14.5,'PRM=',D14.5,'VSMX=',D14.5,'MPSAVE=',D14.5)
040300 CALL SAT(XN,NY,NXY,NXY,XD,YD,D,H,W,S,PR,PHI,FF1,FF22)
040400 WRITE(6,3400) HMIN,NHSAVE
040500 WRITE(6,3400) HMIN,NHSAVE
040600 FORMAT(5X,'HMIN=',D14.5,'NHSAVE=',D14.5)
040700 WRITE(6,810) SUM2
040800 WRITE(6,810) SUM2
040900 FORMAT(6X,'SUM2=',D14.5)
041000 WRITE (10) (PR(N),N=1,NXY)
041100 WRITE (10) H0,(N,N=1,NXY)
041200 REMIND 10
041300 IF(ITER.NE.0.AND.ERR.LT.DEL2) GO TO 32
041400 IF(ITER.GT.ITHAX) GO TO 32
041500 GO TO 22
041600 C APPLIED NORMAL LOAD
041700 C
041800 C
041900 32 CONTINUE
042000 QU4=0.0
042100 PSUM=0.0
042200 DO 37 I=2,NXM1
042300 QUI=2.0
042400 QU3=0.0
042500 IM=I/2
042600 IBM=IM+IM
042700 IF(IBM.NE.1) QUI=1.000
042800 DO 36 J=2,NY
042900 N=I+(J-1)*NX
043000 PSUM=PSUM+PR(N)
043100 QUI=QUI+PR(N)
043200 QU4=QU4+QU3*QUI
043300 FBAR=2.*EPHAX1*HDX1*QU4/(3.*WZC*WZD)
043400 FCHECK=1.*EPHAX1*HDX1*PSUM/(ZC*WZD)

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043500 PBAR=FBAR*PRX/EP
043600 PCHECK=FCHECK*PRX*PRX/EP
043700 WRITE(6,2400) MFITER,FBAR,FCHECK
043800 WRITE(4,2400) MFITER,FBAR,FCHECK
043900 FORMAT(IX,' MFITER=',I3,' FBAR=',D14.6,' FCHECK=',D14.6)
044000 C
044100 C
044200 C
044300 C
044400 C
044500 C
044600 C
044700 C
044800 C
044900 C
1200
2400
AX=(6.0*MXKX*WES*FBAR/(PI*E*PHO))*(1.0/3.0)
BX=AX/XK
CINLT=XD(MCGF1)/BX
COULT=(XD(CNX)-XD(MCGF1))/BX
CSIDE=YD(NHY)/AX
WRITE(6,1200) AX,BX,CINLT,COULT,CSIDE,AX1,BX1
FORMAT(IX,'AX=',D12.5,' BX=',D12.5,' CINLT=',D12.5,
COULT=',D12.5,' CSIDE=',D12.5,' AX1=',D12.5,' BX1=',D12.5)
SUM3=DABS((FP-FBAR)/FP)
IF(SUM3.LE.DEL3)OR.MFITER.GT.MFITMX)GO TO I2
IF(ER.LT.0.030)ITMAX=60
IF(ER.LT.0.010.AND.MFITMX.LT.40)MFITMX=1+MFITMX
IF(SUM3.LT.1.0D-3)DEL2=0.6*DEL2
IF(ITER.GT.132)ROSHI=ROSHI+20.0
IF(ITER.LE.5)XJENN=0.300
IF(ITER.GT.5)XJENN=0.400
IF(ITER.GT.10)XJENN=0.600
IF(ITER.GT.25)XJENN=0.900
HO=HO*(XJENN+(1.0-XJENN)*(FP/FBAR)**ZZZ)
WRITE(6,4400)HO
WRITE(4,4400)HO
FORMAT(IX,'HO=',D14.6)
CONTINUE
DO 440 J=1,NHY
JP=(J-1)*NX
DO 441 I=1,NX
N=I+JP
H(N)=HO+PRX*(S(N)+W(N))
PHI(N)=PR(N)*H(N)*(N+1.5)
CONTINUE
GO TO 11
CONTINUE
STRS=1.5*FBAR/(PI*MX*BX)
HAMR=3.63*GMM0*.68*GMM0*.49*MPBAR*(1.0-DEXP(-0.68*MXK))
HMH=2.69*GMM0*.67*GMM0*.53*MPBAR*(1.0-0.61*DEXP(-0.73*MXK))
99
FRICION FORCE
CONTINUE
DO 450 J=1,NHY
JP=(J-1)*NX
DO 452 I=2,NX*MI
N=I+JP
GPR(N)=(H(N)+CFDP/H(N))*(PR(N+1)-PR(N-1))
CONTINUE
GSUM=0.0
DO 451 J=1,NHY
045000
045100
045200
045300
045400
045500
045600
045700
045800
045900
046000
046100
046200
046300
046400
046500
046600
046700
046800
046900
047000
047100
047200
047300
047400
047500
047600
047700
047800
047900
048000
048100
048200
048300
048400
048500
048600
048700
048800

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048900      N=(J-1)*MX+1
049000      N1=(J-1)*MX+NX
049100      GPR(N)=(H(N)*CFDP/H(N))*(-3.0*PR(N)+4.0*PR(N+1)-PR(N+2))
049200      GPR(N1)=0.0
049300      CONTINUE
049400      MHY1=MHY-1
049500      DO 465 J=1,MHY1
049600      JQR=(MHY+J-1)*MX
049700      JQ=(MHY-J-1)*MX
049800      DO 465 I=1,NX
049900      M1=I+JQR
050000      M2=I+JQ
050100      GPR(M1)=GPR(M2)
050200      CONTINUE
050300      QU4=0.0
050400      PSUM=0.0
050500      DO 460 I=1,NXM1
050600      QU1=2.0
050700      QU3=0.0
050800      QU2=1.0
050900      IM=I/2
051000      IM=IM+IM
051100      IF(I*IM.NE.1) QU1=1.0D0
051200      IF(I.EQ.1) QU2=0.5
051300      DO 461 J=1,NY
051400      N1=(J-1)*MX
051500      QV1=1.0
051600      IF(J.EQ.1.OR.J.EQ.NY) QV1=0.5
051700      PSUM=PSUM+GPR(N)*QV1*QU2
051800      QU3=QU3+GPR(N)*QV1
051900      QU4=QU4+QU3*QU1*QU2
052000      CFRI=AX1*MBX1*HEP*RX*RX*DZC*QU4/(FPM3.0*2.0)
052100      CFRICK=AX1*MBX1*HEP*RX*RX*DZC*PSUM/(FPM2.0*2.0)
052200      WRITE(6,1015) MHQ,H(NCTR),PR(NCTR)
052300      WRITE(4,1015) MHQ,H(NCTR),PR(NCTR)
052400      FORMAT(1X,'MHQ=',D12.5,' H(NCTR)=',D12.5,' PR(NCTR)=',D12.5)
052500      C
052600      C
052700      C
052800      WRITE(6,OUTPUT)
052900      WRITE(6,FRIC1)
053000      WRITE(4,FRIC1)
053100      C
053200      C
053300      C
053400      C
053500      C
053600      C
053700      C
053800      C
053900      C
054000      C
054100      C
054200      C

```

```

054300 2150      FORMAT(1H ,7H DENSITY/30(1H ,10D13.5/))
054400      WRITE(6,2250) (VISC(N),N=1,1,1,NXHY)
054500 2250      FORMAT(1H ,9H VISCOSITY/30(1H ,10D13.5/))
054600      WRITE(6,2350) (XE(N),N=1,1,NHY)
054700 2350      FORMAT(1X ,XE/20(1X,10D13.5/))
054800 43      CONTINUE
054900      WRITE(6,1700) (PRCN),N=1,NXHY)
055000 1700      FORMAT(1H ,2H PR/50(1H,10D13.5/))
055100      WRITE(6,1900) (H(N),N=1,NXHY)
055200 1900      FORMAT (1H ,1H H/50(1H ,10D13.5/))
055300      WRITE(6,1800) (PHI(N),N=1,NXHY)
055400 1800      FORMAT(1H ,3H PHI/50(1H ,10D13.5/))
055500 C      WRITE(6,2000) (S(N),N=1,NXHY)
055600 2000      FORMAT (1H ,1H S/50(1H ,10D13.5/))
055700 C      WRITE(6,2100) (DENS(N),N=1,NXHY)
055800 2100      FORMAT(1H ,7H DENSITY/50(1H ,10D13.5/))
055900      WRITE(6,2200) (VISC(N),N=1,NXHY)
056000 2200      FORMAT(1H ,9H VISCOSITY/50(1H ,10D13.5/))
056100 C
056200 53      CONTINUE
056300      WRITE(8) NX,NHY,NXHY,(PRCN),N=1,NXHY),CXD(N),N=1,NX),CYD(N),N=1,NHY),PRMX
056400      MCGF2=MCGF1+NZOH2
056500      WRITE(8)NXY,MCGF1,MCGF2,JZON,(H(N),N=1,NXHY),RX,RY,AX,BX
056600 63      CONTINUE
056700      STOP
056800      END

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```

0000020 C
0000100 ASHRI
0000200 SUBROUTINE SAT(NX,NY,MNY,NXY,MXNY,XD,YD,D,M,W,S,PR,PHI,FF1,FF22)
0000300 IMPLICIT REAL*8(A-M,G-Z)
0000400 DIMENSION D(NX,NY,9)
0000500 DIMENSION PR(NXY),PHI(NXNY),H(NXNY),M(NXNY),S(NXNY)
0000600 DIMENSION FF1(NXNY),FF22(NXNY)
0000700 DIMENSION XD(NX),YD(NY)
0000800 COMMON/AA/MO,PRX,MMIN,MNSAVE
0000900 DATA PI/3.1415926535900/
0001000 MNYM1=MNY-1
0001100 DO 5 J=1,MNYM1
0001200 JQ=(MNY+J-1)*MX
0001300 JQ=(MNY-J-1)*MX
0001400 DO 5 I=1,NX
0001500 M1=I+JQ
0001600 M2=I+JQ
0001700 PR(M1)=PR(M2)
0001800 CONTINUE
0001900 DO 10 J=2,NY,2
0002000 JP=(J-1)*MX
0002100 N=I+JP
0002200 IF(PR(M).EQ.0.0) GO TO 20
0002300 M12=M-1
0002400 M32=M+1
0002500 M11=M12-MX
0002600 N21=M11+1
0002700 N31=M11+2
0002800 M13=M12+MX
0002900 N23=M13+1
0003000 N33=M13+2
0003100 DO 30 JM=1,MNY
0003200 JP=(JM-1)*MX
0003300 JD=J-JM
0003400 JN=IAB5(JD)+1
0003500 DO 40 IN=1,NX
0003600 M=IM+JMP
0003700 PW=W(M)
0003800 ID=I-ID
0003900 IN=IAB5(ID)+1
0004000 IF(ID.GE.0.AND.JD.GE.0) GO TO 102
0004100 IF(ID.LT.0.AND.JD.LT.0) GO TO 103
0004200 IF(ID.GE.0.AND.JD.LT.0) GO TO 104
0004300 GO TO 105
0004400 K12=M12
0004500 K13=M13
0004600 K21=M21
0004700 K22=M22
0004800 K23=M23
0004900 K31=M31
0005000 K32=M32
0005100 GO TO 101

```


105200	103	K12=M32	
105300		K12=M12	
105400		K23=M21	
105500		K13=M31	
105600		K33=M11	
105700		K21=M23	
105800		K11=M33	
105900		K31=M13	
106000		GO TO 101	
106100	104	K12=M12	
106200		K12=M32	
106300		K23=M21	
106400		K13=M11	
106500		K33=M31	
106600		K21=M23	
106700		K11=M13	
106800		K31=M33	
106900			
107000		K12=M32	
107100		K32=M12	
107200		K23=M23	
107300		K13=M33	
107400		K33=M13	
107500		K21=M21	
107600		K11=M31	
107700		K31=M11	
107800	101	CONTINUE	
107900		W22=DCIM,JN,9)=PR(N)	
108000		W12=DCIM,JN,4)=PR(K12)	
108100		W32=DCIM,JN,6)=PR(K32)	
108200		W23=DCIM,JN,8)=PR(K23)	
108300		W13=DCIM,JN,7)=PR(K13)	
108400		W33=DCIM,JN,9)=PR(K33)	
108500		W21=DCIM,JN,2)=PR(K21)	
108600		W11=DCIM,JN,1)=PR(K11)	
108700		W31=DCIM,JN,3)=PR(K31)	
108800		W(N)=(W22+W12+W32+W23+W13+W33+W21+W11+W31)+PM	
108900		CONTINUE	
109000	40	CONTINUE	
109100	30	CONTINUE	
109200	20	CONTINUE	
109300	10	CONTINUE	
109400	C	FILM THICKNESS	
109500	C	MMIN=1.0	
109600	C	DCFI=1.0/(60.0*PI)	
109700		DO 55 J=1,NM	
109800		JP=(J-I)*MX	
109900		DO 55 I=1,NX	
101000		N=JP+I	
101100		W(N)=DCFTW(N)	
101200		H(N)=HO+(S(N)+W(N))*PRX	
101300		PDM=1.0/(H(N)*H(N))	
101400			
101420			

```

10440
10460
10500
10600
10700
10800 51
10900 55
11000 C
11100 C
11200 C400
11300
11400

FF11(N)=1.0+CF11*PDH
FF22(N)=1.0+CF22*PDH
IF(H(N).GT.HMIN) GO TO 51
HMIN=H(N)
NMSAVE=N
PHI(N)=PR(N)*H(N)*W1.5)
CONTINUE
WRITE(4,400) NMSAVE,HMIN
WRITE(6,400) NMSAVE,HMIN
FORMAT(IX,'NMSAVE=',I10,'
RETURN
END
HMIN=',D16.5)

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16. Abstract <p>Minimum film thickness results for piezoviscous-rigid regime of lubrication are developed for a compressible Newtonian fluid with Roelands viscosity. The results provide a basis for the analysis and design of a wide range of machine elements operating in the piezoviscous-rigid regime of lubrication. A new numerical method of calculating elastic deformation in contact stresses is developed using a biquadratic polynomial to approximate the pressure distribution on the whole domain analyzed. The deformation of every node is expressed as a linear combination of the nodal pressures whose coefficients can be combined into an influence coefficient matrix. This approach has the advantages of improved numerical accuracy, less computing time and smaller storage size required for influence matrix. The ideal elastohydrodynamic lubrication is extended to real bearing systems in order to gain an understanding of failure mechanisms in machine elements. The improved elastic deformation calculation is successfully incorporated into the EHL numerical scheme. Using this revised numerical technique and the flow factor model developed by Patir and Cheng (1978) the surface roughness effects on the elastohydrodynamic lubrication of point contact is considered. Conditions typical of an EHL contact in the piezoviscous-elastic regime entrained in pure rolling are investigated. Results are compared with the smooth surface solutions. Experiments are conducted to study the transient EHL effects in instrument ball bearings. Results indicate a subregime of elastohydrodynamic lubrication called parched. A system is parched if oil films outside the Hertz contacts are so thin they do not flow under service accelerations.</p>					
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